

# SEMICONDUCTOR DETECTOR

A semiconductor detector is a ionizing radiation detector.

It is a reverse biased junction diode in which the passage of the nuclear radiation creates the electron hole pairs in the semiconductor materials & are collected by the applied electric field.

Silicon and germanium are the two basic semiconducting materials which are used for the detectors. The important properties of semiconductor which let them to use as detectors are the band gap, net impurity doping and life time of free charge carriers.

The drift velocity of electron and holes are given by

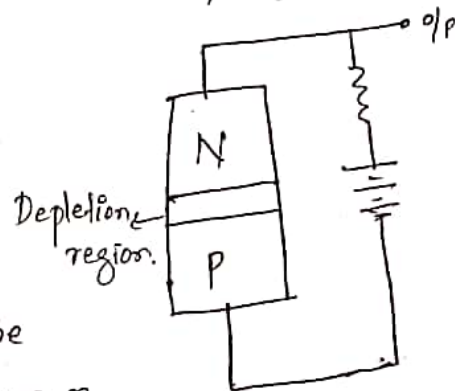
$$v_e = \mu_e E \quad v_h = \mu_h E$$

where  $\mu_e$  &  $\mu_h$  are the mobilities of electrons & holes  
 $E$  = Applied electric field.

## CONSTRUCTION AND WORKING

Semiconductor detector is a reverse biased p-n junction diode detector as shown in the figure.

The p-n junction detector is reverse biased in order to increase the width of the depletion layer in between the p-side and n-side of the p-n junction detector. The depletion region



is the active region. When the ionizing radiation enters the depletion layer, electrons and holes pairs are created. The electrons and holes are collected by the respective electrodes. Their movement of the electric charges across the p-n junction constitutes the electric current. This electric current is converted into voltage pulse. The voltage is amplified and analysed using suitable electronic device.

The thickness of the depletion region is given by the eqn.

$$d = \left( \frac{q \epsilon_0 \epsilon_r (V_b \pm V_a)}{e} \right)^{1/2} \left( \frac{N_d + N_a}{N_a N_d} \right)^{1/2}$$

positive sign for forward bias voltage and negative for reverse bias voltage.

where  $k$  - Ratio of the permittivity of the material to that of the vacuum

$\epsilon_0$  - permittivity of the free space.

$V_b$  - applied potential

$V_a$  - Diffusion (or) junction potential.

$e$  - electronic charge.

$N_A$  - Number of acceptor atoms per unit volume.

$N_D$  - Number of donor atoms per unit volume.

# NUCLEAR MODELS

# ANALOGY BETWEEN A SMALL LIQUID DROP AND A NUCLEUS

- liquid drop is spherical in shape due to surface tension force acting towards the centre. the nucleus is also assumed to be spherical in shape.
- density of spherical drop is known to be independent of its volume. nuclear density is also found to be independent of nuclear volume

- the molecules of a liquid drop are in continual random motion. extending these analogy to nucleus, due to random collisions between various nucleons, these may pick up sufficient energy and escape. the model explains nuclear reactions and nuclear fission.

## ■ LIQUID DROP MODEL

- this model was proposed by niels bohr. this model is based upon the fact that molecules in a liquid are held together by short range intermolecular forces known as coherent forces. density of liquid drop is very high.

# ASSUMPTIONS

- all nuclei are considered to behave like incompressible drop of liquid.
- nucleus is spherical in shape due to symmetrical surface tension forces.
- density of liquid is constant and independent of size or shape of liquid drop.
- binding energy per nucleon is constant just as latent heat of vaporization is constant

- binding energy per nucleon is proportional to mass number a just as energy required to evaporate liquid is proportional to its mass.

semi –empirical mass formula

total binding energy of a nucleus is given by

$$B.E = [ZM_p + (A-Z)M_n - M]_{\text{NUCLEUS}} - B/C^2$$



- NUCLEAR MASS IS GIVEN BY

$$M_{\text{NUCLEUS}} = [ZM_p + (A-Z) M_N] - B/C^2$$

THIS IS CALLED SEMI-EMPIRICAL MASS  
FORMULA.

TOTAL BINDING ENERGY

$$E_B = E_V + E_S + E_C + E_A + E_P$$

## ■ 1) volume energy

binding energy per nucleon has constant value over wide range of mass numbers.

so we have  $E_v = a_v A$  where  $a_v$  is a constant

## . 2) surface energy

surface nucleons have fewer neighbors than nucleons lying deep inside the nucleus. so binding energy reducec. this decrease in binding energy is proportional to the surface area of nucleus.

- $E_s = -a_s A^{2/3}$
- negative sign indicates that this energy has to be deducted from the total energy.

- **3)coulomb energy**

coulomb energy of a nucleus is proportional to the potential energy of  $Z$  protons packed together in symmetric assembly of radius  $r$

- $E_C = -a_c Z(Z-1)A^{-1/3}$

#### 4) asymmetry energy

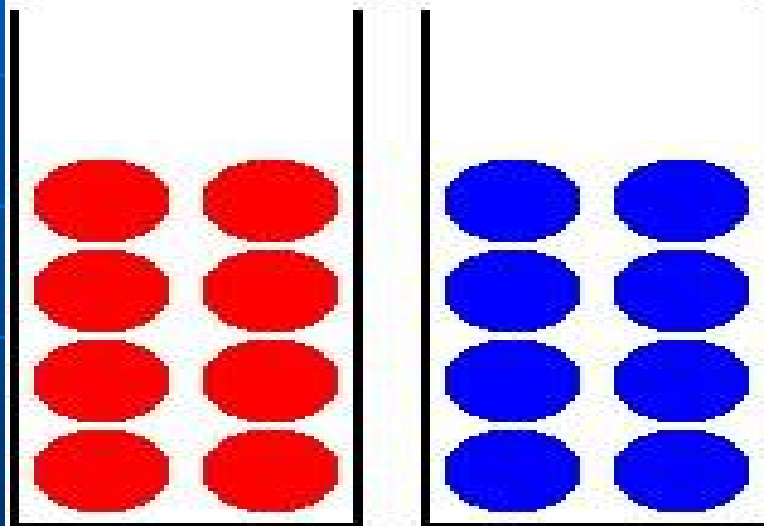
for heavy nuclei, as no. of neutrons increases, nucleus acquires an asymmetrical character, due to which a force comes into play which reduces the asymmetry energy

$$E_A = -a_a (A - 2Z)^2 / A$$

$a_a$  is a constant

$$A = 16$$

Lower energy

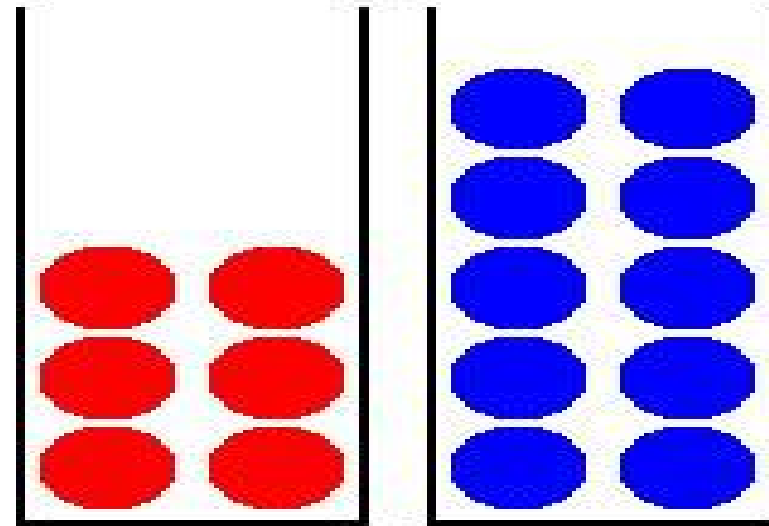


Protons

Neutrons

$$|N - Z| = 0$$

Higher energy



Protons

Neutrons

$$|N - Z| = 4$$

- pairing and shell energy

nuclei with even  $z$  and even  $n$  are highly stable, even  $z$ - odd  $n$ , or odd  $z$ -even  $n$  are less stable. so this effect contributes towards volume energy by

$$E_p = a_p A^{-3/4}$$

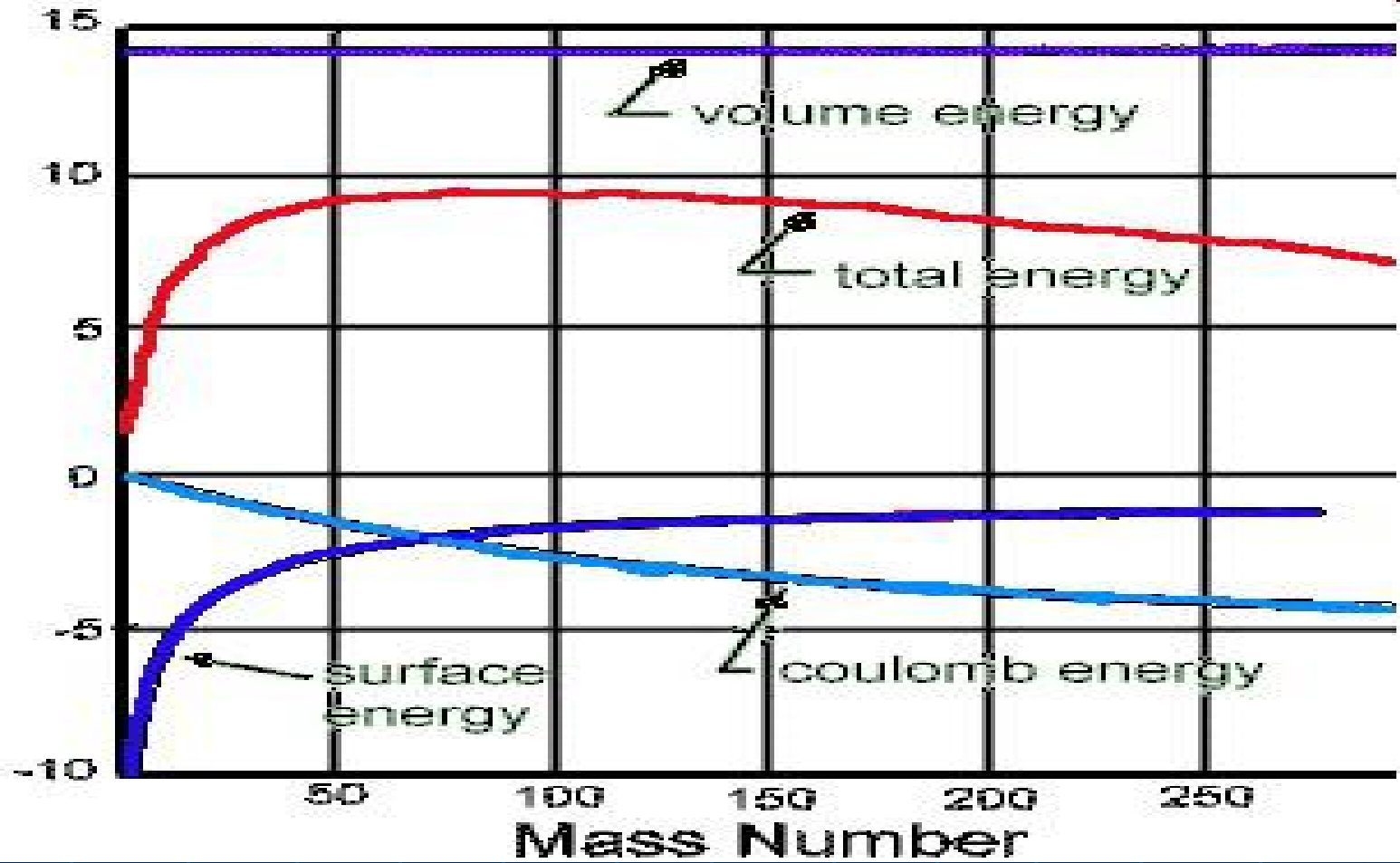
where  $a_p$  is constant

combining all these terms we get total binding energy  $E_B = E_V + E_S + E_C + E_A + E_p$

- so binding energy per nucleon is given by

$$E_B/A = A_V - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_a (A - 2Z)^2/A + a_p A^{-3/4}$$

Binding Energy per Nucleon (MeV)





# ACHEIVEMENTS

- **1) STABLE NUCLEUS:-**

stability of liquid drop is due to force of cohesion between the molecules. and stability of nucleus is due to the binding energy of each nucleon.

- **2) RADIOACTIVE NUCLEUS:-**

a liquid evaporates by gaining energy from its neighboring molecules during the process of collisions. Similarly nucleon may leave the nucleus by gaining energy from neighboring nucleons, thus exhibiting the process of radioactivity

## ■ **ARTIFICIAL RADIOACTIVITY:-**

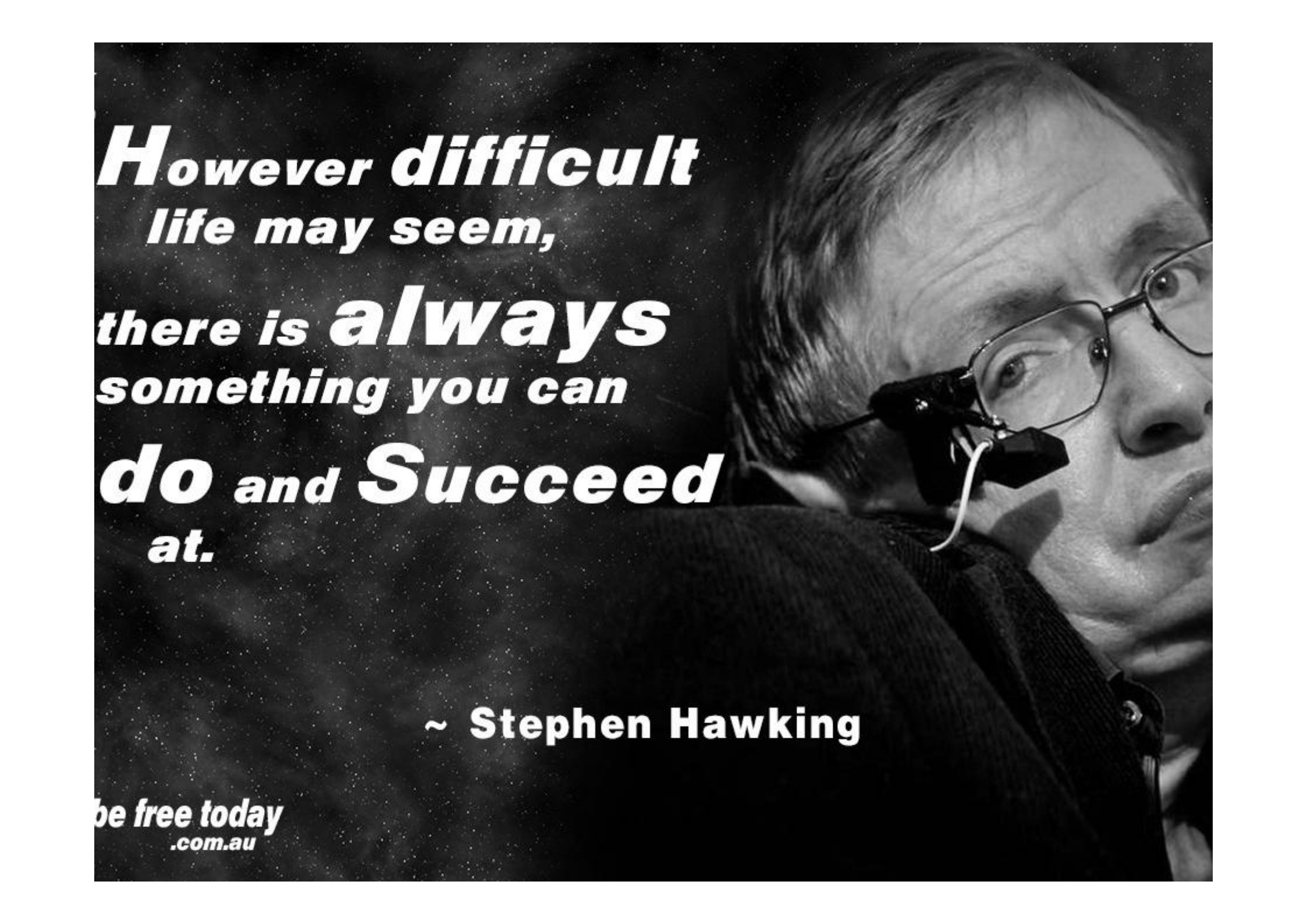
liquid drop model also explains the phenomenon of artificial radioactivity

## **FISSION:-**

liquid drop model also explains the phenomenon of nuclear fission

## **FAILURES:-**

- 1)it fails to explain the high stability of nuclei with magic numbers
- 2)it fails to explain the measured spin and magnetic moments of the nuclei

A black and white close-up photograph of Stephen Hawking. He is wearing his characteristic glasses and a speech synthesizer device is attached to his neck. He has a thoughtful expression, looking slightly to the right. The background is dark and textured, resembling a starry night sky.

***However difficult  
life may seem,  
there is **always**  
something you can  
**do** and **Succeed**  
at.***

**~ Stephen Hawking**

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# Sunitha williams



## DIFFERENCE BETWEEN SHELL MODEL AND LIQUID DROP MODEL

- in liquid drop model, it is assumed that nucleons interact strongly with immediate neighbors.
- but shell model treats nucleons individually and it is assumed that nucleons do not interact with each other

## EVIDENCE IN FAVOUR OF SHELL MODEL

- nucleons have a tendency to form pair and its difficult to remove a paired nucleon than unpaired.
- nuclei with even  $Z$  and even  $N$  are most abundant, odd  $Z$  and odd  $N$  are least abundant, even  $N$  and odd  $Z$  or even  $Z$  and odd  $N$  come in between

- It has been found that the nuclei with proton number or neutron number equal to certain numbers 2,8,20,28,50,82 and 126 behave in a different manner when compared to other nuclei having neighboring values of  $Z$  or  $N$ . Hence these numbers are known as MAGIC NUMBERS

- 

- Nuclei with magic numbers of neutrons or protons have their first excited states at higher energies than in cases of the neighboring nuclei

# ASSUMPTIONS

- nucleons form closed sub-shells within the nucleus just as electrons in case of atoms
- nucleons in nucleus are arranged in shell structure. The shells get closed with suitable no. of protons and neutrons
- each nucleon moves independently inside a nucleus in a fixed orbit under the effect of central potential produced by average interaction between remaining  $(A-1)$  nucleons in it.



- each nucleon is assumed to possess a spin angular momentum and orbital angular momentum

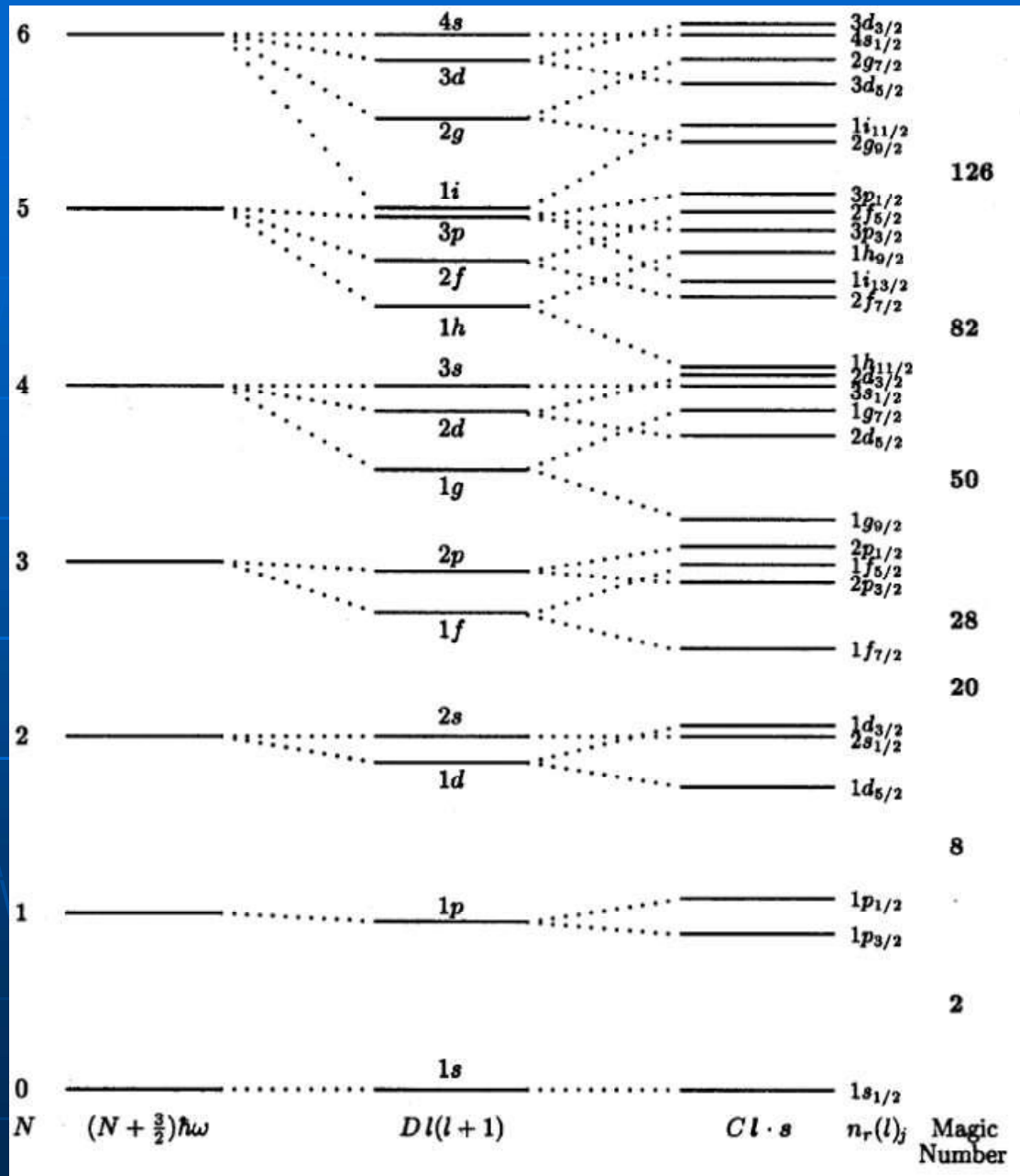
- Consider a nucleon moving independently in the harmonic oscillator potential which is spherically symmetric. The Schrodinger equation given below can be solved in the Cartesian coordinate system as well as in the spherical coordinate system.

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r})\right)\psi(\mathbf{r}) = E\psi(\mathbf{r}),$$

$$V(\mathbf{r}) = \frac{1}{2}m\omega^2 r^2 = \frac{1}{2}m\omega^2(x^2 + y^2 + z^2).$$

$$E = \hbar\omega \left( n_x + n_y + n_z + \frac{3}{2} \right) = \hbar\omega \left( N + \frac{3}{2} \right),$$

$N$	$E$	Orbitals ( $n_r, l$ )	$\mathcal{N}_N = (N+1)(N+2)$ $= \sum_l 2(2l+1)$	$\sum_N \mathcal{N}_N$
0	3/2	1s	2	2
1	5/2	1p	6	8
2	7/2	1d, 2s	12	20
3	9/2	1f, 2p	20	40
4	11/2	1g, 2d, 3s	30	70
5	13/2	1h, 2f, 3p	42	112
6	15/2	1i, 2g, 3d, 4s	56	168



# Success of shell model

- magic numbers:-shell model explains the existence of magic no.s

it has been found that the nuclei with proton number or neutron number equal to certain numbers 2,8,20,28,50,82 and 126 behave in a different manner when compared to other nuclei having neighboring values of  $Z$  or  $N$ . hence these numbers are known as magic numbers.

- SPIN :-Shell model successfully explains the ground state spins and magnetic moments of the nuclei.
- even-even nuclides (both  $Z$  and  $A$  even) have zero intrinsic spin and even parity.
- odd  $A$  nuclei have one unpaired nucleon. the spin of the nucleus is equal to the  $j$  value of that unpaired nucleon

- **MAGNETIC MOMENT:-** Shell model successfully explains the magnetic moments of the nuclei.
- For even-even nuclei magnetic moment is zero
- For even-odd or odd-even magnetic moment depends upon the last unpaired nucleon whether its proton or a neutron.

- STABILITY:-VERY HIGH STABILTY AND HIGH BINDING ENERGY IS ALSO EXPLAINED ON THE BASIS OF CLOSED SHELLS.
- NUCLEAR ISOMERISM:-EXISTENCE OF ISOBARIC,ISOTOPIC NUCLEI IN DIFFERENT ENERGY STATE IS ALSO EXPLAINED ON THE BASIS OF SHELL MODEL



# MAGIC NUMBERS

- It has been found that the nuclei with proton number or neutron number equal to certain numbers 2, 8, 20, 28, 50, 82 and 126 behave in a different manner when compared to other nuclei having neighboring values of  $Z$  or  $N$ . Hence these numbers are known as magic numbers.

- **Experimental evidences for the existence of magic numbers;**
- 1. The binding energy of magic numbered nuclei is much larger than the neighboring nuclei. Thus larger energy is required to separate a single nucleon from such nuclei.
- 2. Number of stable nuclei with a given value of  $Z$  and  $N$  corresponding to the magic number are much larger than the number of stable nuclei with neighboring values of  $Z$  and  $N$ . For example, Sn with  $Z=50$  has 10 stable isotopes, Ca with  $Z=20$  has six stable isotopes.

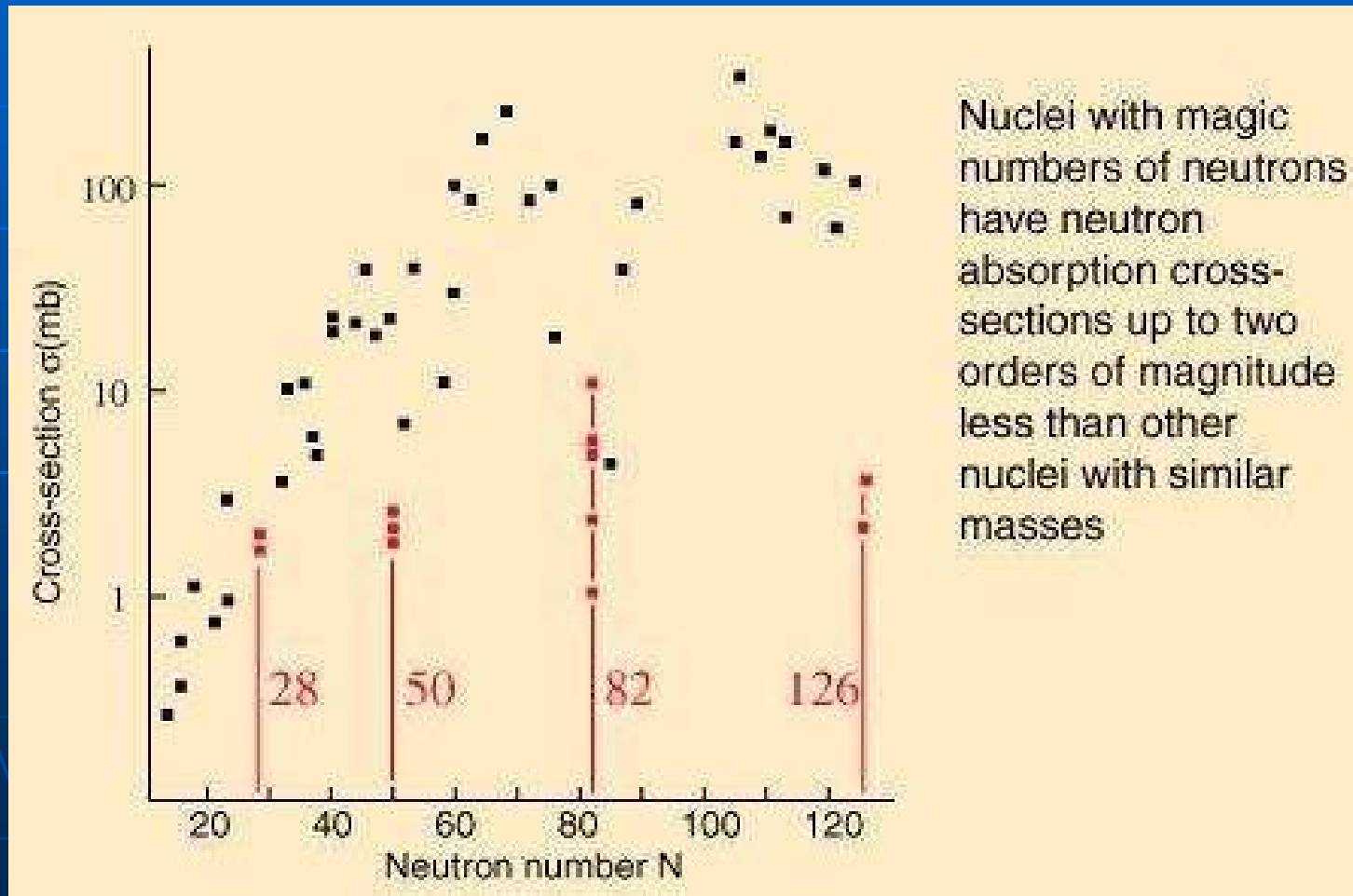
- 3. Naturally occurring isotopes whose nuclei contain magic numbered  $Z$  or  $N$  have greater relative abundances. For example, Sr-88 with  $N=50$ , Ba-138 with  $N=82$  and Ce-140 with  $N=82$  have relative abundances of 82.56%, 71.66% and 88.48% respectively.
- 4. Three naturally occurring radioactive series decay to the stable end product Pb with  $Z=82$  in three isotopic forms having  $N=126$  for one of them.
- 5. Neutron absorbing cross section is very low for the nuclei having magic numbered neutron number.

6. Nuclei with the value of  $N$  just one more than the magic number spontaneously emit a neutron (when excited by preceding beta-decay) E.g., O-17, K-87 and Xe-137.
7. Nuclei with magic numbers of neutrons or protons have their first excited states at higher energies than in cases of the neighboring nuclei.
8. Electric quadrupole moment of magic numbered nuclei is zero indicating the spherical symmetry of nucleus for closed shells.
9. Energy of alpha or beta particles emitted by magic numbered radioactive nuclei is larger than that from other nuclei.

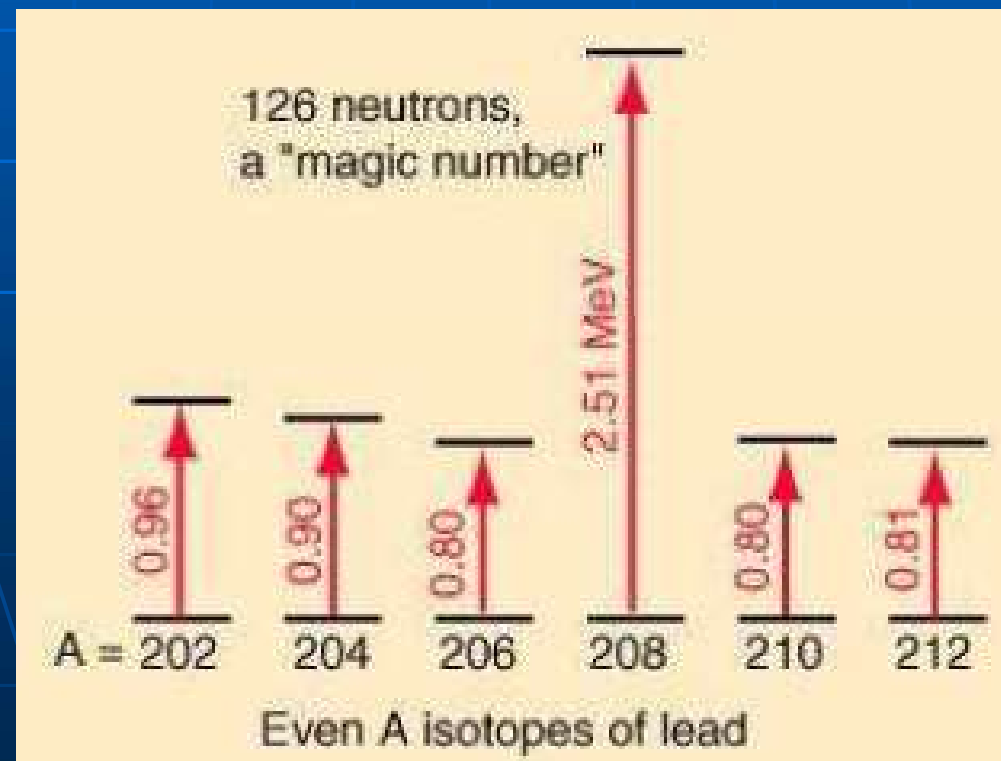
# SPECIAL FEATURES OF MAGIC NUCLEI

- The neutron (proton) separation energies (the energy required to remove the last neutron(proton)) peaks if  $N$  ( $Z$ ) is equal to a magic number.
- There are more stable isotopes if  $Z$  is a magic number, and more stable isotones if  $N$  is a magic number

- If  $N$  is magic number then the cross-section for neutron absorption is much lower than for other nuclides.



- The energies of the excited states are much higher than the ground state if either N or Z or both are magic numbers.



# Fermi gas model

- It is a **statistical model** of the nucleus
- This model pictures the nucleus as a **degenerate gas of protons and neutrons** much like the **free electron gas in metals**
- The gas is considered degenerate **because all the particles are crowded into the lowest possible states** in a manner consistent with the requirement of **Pauli exclusion principle**



**FERMIONS** are subatomic particles obey  
fermi - derac statistics

- Nucleons are **fermions** having **spin  $\frac{1}{2}$** . Thus the **behaviour of the neutron or the proton gas** will be determined by **Fermi-Dirac statistics**
- In such a gas at **0 K**, all the **energy levels upto a maximum**, known as **Fermi energy  $E_F$**  are occupied by the particles, each level being occupied by **two particles with opposite spins**.

Bosons are subatomic particles obeys  
Bose-Einstein's statistics

- The nucleons move freely within a spherical potential well of the proper diameter with depth adjusted so that the Fermi energy raises the highest lying nucleons upto the observed binding energies.

- The **potential well** is filled separately with **nucleons of each type**, allowing just two particles of a given type with **opposite spin** to each cell in phase space of volume  $h^3$ .

According to **Fermi-Dirac statistics**,

- the number of **neutron states per unit momentum** interval is  $\frac{dN}{dp} = \frac{2 \times 4\pi p^2 V}{(2\pi\hbar)^2} = \frac{V p^2}{\pi^2 \hbar^3}$
- where  $V$  is the volume of the nucleus.

- If  $p_f$  is the **limiting momentum** below which all the states are filled.

$$p_f = \sqrt{2m E_f}$$

the number of neutrons occupying momentum states up to this maximum momentum is obtained as

$$N = \int_0^{p_f} \frac{dN}{dp} dp = \frac{V}{3\pi^2 \hbar^3} p_f^3$$

$$p_f = (3\pi^2)^{\frac{1}{3}} \hbar \left( \frac{N}{V} \right)^{\frac{1}{3}}$$

$$E_f = \frac{p_f^2}{2M} = (3\pi^2)^{\frac{2}{3}} \frac{\hbar^2}{2M} \left( \frac{N}{V} \right)^{\frac{2}{3}}$$

$$= \frac{\hbar^2}{2M} \left( \frac{3\pi^2 N}{V} \right)^{\frac{2}{3}}$$

$$V = \frac{4}{3} \pi r_0^3 A$$

- where  $V$  is the nuclear volume which contains  $N$  particles (fermions) and  $M$  is the nucleonic mass.
- We have **two** different types of **Fermi gas** in the nucleus (i) the **proton gas** and (ii) the **neutron gas**. The respective numbers of **protons** and **neutrons** are  $Z$  and  $A - Z$ . Now, assuming that the number of nucleonic states to be equal to the nucleon number in each case, one obtains the density of states for the two gases as

the density of states for the two gases

$$n_p = \frac{Z}{V} = \frac{Z}{\frac{4}{3}\pi r_0^3 A} = \frac{3Z}{4\pi r_0^3 A}$$

$$n_n = \frac{A-Z}{V} = \frac{3(A-Z)}{4\pi r_0^3 A}$$

We have  $r_0 = 1.2$  fm. One obtains after assuming  $N = A - Z = \frac{A}{2}$

$$n_p = n_n = \frac{\frac{3}{2}}{4\pi(1.2)^3} = 0.069 \frac{\text{nucleons}}{\text{m}^3}$$

$\therefore$  Nucleon density  $N = n_p + n_n = 0.138 \frac{\text{nucleons}}{\text{m}^3}$

# Fermi energy

- each state can be occupied by nucleons of opposite spins and substituting

$$E_f = \frac{\hbar^2}{2M} \left( \frac{3}{2} \pi^2 n_p \right)^{\frac{2}{3}} = 21 \text{ MeV}$$



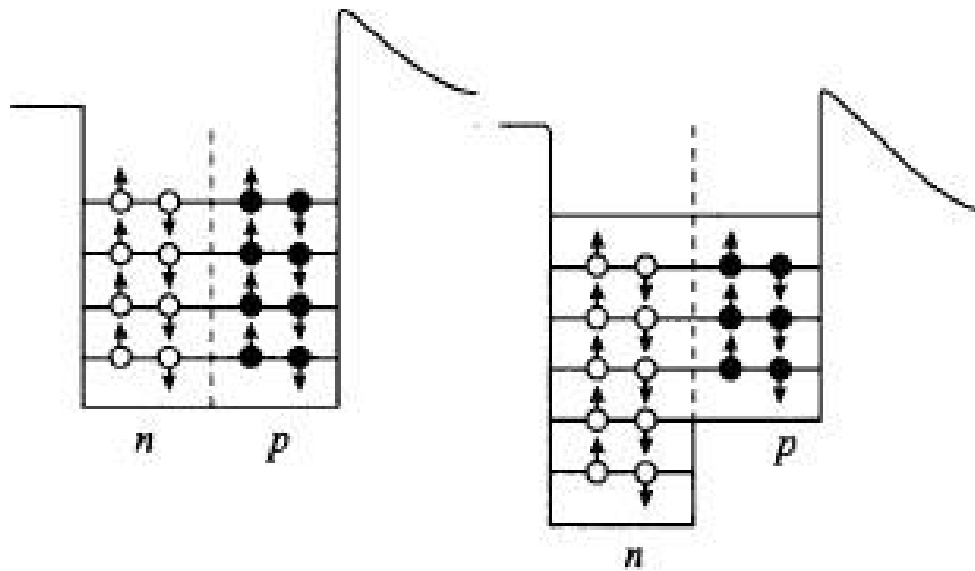
- However, the number of protons ( $Z$ ) and neutrons ( $A - Z$ ) in an actual nucleus are not equal and hence  $N$  being somewhat greater than  $Z$ . Obviously, the Fermi energies of the two types of nucleons are different. Now  $N > Z$ ,  $(E_f)_n > (E_f)_p$  and hence the potential wells for the protons and neutrons have different depths, i.e. the former being less deep than the latter.

- The depth of the potential well is obtained as

$$V_0 = E_f + \frac{E_B}{A} = E_f + f_B$$

- Here  $f_B = E_B/A$  is called the mean binding energy per nucleon (binding fraction) and is of the order of 8 MeV/nucleon for both protons and neutrons. Figure 1(b) exhibit Fermi gas model of nucleonic potential wells.

Figure exhibits the difference in the depths of wells for neutron and proton. Fig:1



- From Figure 1, we note that the Fermi energies for both protons and neutrons are represented by the same horizontal line, corresponding to about 8 MeV below the top of the potential well (Coulomb effect is neglected). One can visualize this that if these are at different depths below the top of the well, then the nucleons of one type from the higher Fermi level (say, neutrons) would make spontaneous transitions to the lower Fermi level for the other type (protons) by beta transformations. Obviously, the levels would ultimately equalize.

- Thus one finds the depth of the potential well approximately,
- $V_0 \sim 21 + 8 = 29 \text{ MeV}$
- We must note that the neutron depth is slightly greater than the proton depth.
-

- Let us now consider a hypothetical infinite medium of nuclear matter of uniform density in which the numbers of neutrons and protons are equal, i.e.  $N = Z$  and the Coulomb interaction of the proton is considered negligible. In this situation, one obtains from the semi-empirical binding energy Bethe-Weizsacker relation

$$\left( \frac{E_B}{A} \right) = \left( \frac{E_0}{A} \right) = 15.9 \text{ MeV/nucleon}$$

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- Where  $EB$  is binding energy. Adding this to the depth of the potential well below the Fermi level, one obtains the depth of the potential well as
- $V_0 = 21 + 15.9 = 36.9 \text{ MeV/nucleon}$

# Entropy relation

- One can also obtain the energy density of the nuclear levels for a given excitation energy using the entropy relation:  $S = k \ln W$  and thermodynamic expression for entropy.



# conclusion

- One cannot predict detailed properties of low lying states of nuclei observed in the radioactive decay processes from this model. This model is particularly useful in describing phenomena which are sensitive to the high momentum part of the nucleon spectrum. The model suggests that nucleon collisions often do not transfer small amount of momentum to the nucleus, because the nucleon momentum states near the origin are filled. However, this limitation does not affect collisions in which large momentum transfer takes place. Obviously, this statistical model helps to explain the properties of the nucleus in excited states. One can also treat the unbound states of heavy and medium nuclei with the help of this model.

## FERMI GAS MODEL

It is a statistical model of the nucleus. This model pictures the nucleus as a degenerate gas of protons and neutrons much like the free electron gas in metals. The gas is considered degenerate because all the particles are crowded into the lowest possible states in a manner consistent with the requirement of Pauli exclusion principle. In this case the nature of the microscopic particles is fully reflected in its effect on the ensemble as a whole.

Nucleons are fermions having spin  $\frac{1}{2}$ . Thus the behaviour of the neutron or the proton gas will be determined by Fermi-Dirac statistics. In such a gas at

0 K, all the energy levels upto a maximum, known as Fermi energy  $E_f$  are occupied by the particles, each level being occupied by two particles with opposite spins.

Neglecting for the moment, the electrostatic charge of the protons and

supposing that the nucleus has  $N = Z = A/2$ . The nucleons move freely within a spherical potential well of the proper diameter with depth adjusted so that the Fermi energy raises the highest lying nucleons upto the observed binding energies. The potential well is filled separately with nucleons of each type, allowing just two particles of a given type with opposite spin to each cell in phase space of volume  $h^3$ . According to Fermi-Dirac statistics, the number of neutron states per unit momentum interval is

$$\frac{dN}{dp} = \frac{2 \times 4\pi p^2 V}{(2\pi\hbar)^2} = \frac{V p^2}{\pi^2 \hbar^3}$$

$$p_f \left[ = (2M E_f)^{\frac{1}{2}} \right]$$

where  $V$  is the volume of the nucleus. If is the limiting momentum below which all the states all filled. Obviously, the number of neutrons occupying momentum states upto this maximum momentum is obtained as

$$N = \int_0^{p_f} \frac{dN}{dp} dp = \frac{V}{3\pi^2 \hbar^2} p_f^3$$

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$$\therefore \text{Nucleon density} \quad N = n_p + n_n = 0.138 \frac{\text{nucleons}}{\text{m}^3}$$

Remembering that each state can be occupied by nucleons of opposite spins and substituting the above in (3), one obtains

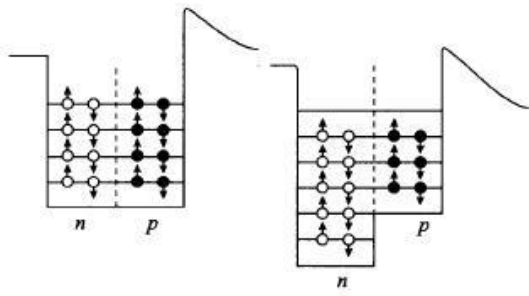
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In this situation, one obtains from the semi-empirical binding energy Bethe-Weizsacker relation

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Where  $E_B$  is binding energy. Adding this to the depth of the potential well below the Fermi level, one obtains the depth of the potential well as

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One expect that any successful theory of nuclear matter should be able to correlate the above value of  $V_0$  to the nature of internucleon nuclear force.

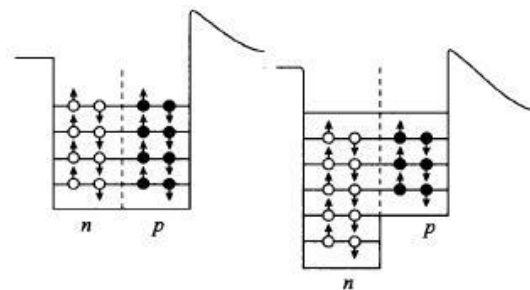
We have so far assumed nuclear temperature to be  $T = 0 \text{ K}$  corresponding to the ground state. When some excitation energy is supplied to the nucleus, then the thermal energy of the nucleus corresponds to  $T > 0 \text{ K}$ . In this case, one can easily show that the total excitation energy is

$$E_t = E_p + E_n = 11(kT)^2 \text{ MeV}$$

Since  $kT \sim 1$  and hence  $E_t \sim 11$  MeV.

One can also obtain the energy density of the nuclear levels for a given excitation energy using the entropy relation:  $S = k \ln W$  and thermodynamic expression for entropy.

One cannot predict detailed properties of low lying states of nuclei observed in the radioactive decay processes from this model. This model is particularly useful in describing phenomena which are sensitive to the high momentum part of the nucleon spectrum. The model suggests that nucleon collisions often do not transfer small amount of momentum to the nucleus, because the nucleon momentum states near the origin are filled. However, this limitation does not affect collisions in which large momentum transfer takes place. Obviously, this statistical model helps to explain the properties of the nucleus in excited states. One can also treat the unbound states of heavy and medium nuclei with the help of this model.



## Degenerate fermions gas Nucleons in a nucleus

Let's apply these results to the system of nucleons in a large nucleus (both protons and neutrons are fermions). In heavy elements, the number of nucleons in the nucleus is large and statistical treatment is a reasonable approximation. We need to estimate the density of protons/neutrons in the nucleus. The radius of the nucleus that contains  $A$  nucleons:

$$R = (1.3 \times 10^{-15} \text{ m}) \times A^{1/3}$$

Thus, the density of nucleons is:

$$n = \frac{A}{\frac{4}{3}\pi (1.3 \times 10^{-15} \text{ m})^3 \times A} \approx 1 \cdot 10^{44} \text{ m}^{-3}$$

For simplicity, we assume that the # of protons = the # of neutrons, hence their density is

$$n_p = n_n \approx 0.5 \cdot 10^{44} \text{ m}^{-3}$$

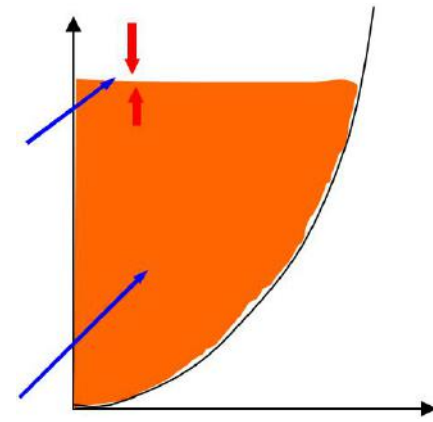
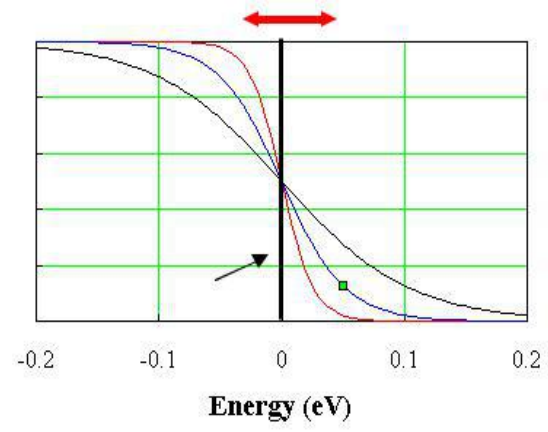
The Fermi energy

$$E_F = \frac{(6.6 \cdot 10^{-34})^2}{8 \times 1.6 \cdot 10^{-27}} \frac{3}{\pi} 0.5 \cdot 10^{44} \text{ }^{2/3} \text{ J} = 4.3 \cdot 10^{-12} \text{ J} = 27 \text{ MeV}$$

**$E_F \gg k_B T$**  – the system is strongly degenerate. The nucleons are very “cold” – they are all in their ground state!

The average kinetic energy in a degenerate Fermi gas = 0.6 of the Fermi energy

$$E = 16 \text{ MeV} \quad - \text{ the nucleons are non-relativistic}$$



# Nuclear shell model

- It is one of the most important and useful model of nuclear structure.
- Its conceptions and name come from the results of empirical correlations of certain nuclear data.
- It is attempt to account for the existance of magic numbers
- Other nuclear properties in terms of nuclear behaviour in common force field



# Assumptions

- 1. Nucleons move freely in nuclei similar to electronic motions in atoms other words nucleons must largely move and behave independently.
- 2. Each nucleon moves in a orbit characterised by a definite angular momentum and definite energy and that it moves independently of other nucleons

# Assumptions conti..

- 3.The orbit is determined by potential energy function  $V(r)$  which represents the average effect of the interactions with other nucleons and is same for each particle.
- 4.each nucleon is regarded as independent particle and interactions between nucleons is considered to be a small perturbation on interaction between nucleon and potential field. Hence it is also called independent particle model.

# Assumptions conti..

- Potential energy analogous to the coulomb energy and orbit of nucleon is analogous to electron orbit.
- 5. Most of the nucleons are paired so that a pair of nucleons contributes zero spin and zero magnetic moment.
- 6. paired nucleons form inert core (does not move).

# Assumptions conti..

- Properties of odd  $A$  nuclei are characterised by unpaired nucleon and odd-odd nuclei by unpaired proton and neutron.
- Nucleons form closed shells.

# PREDICTIONS

- 1. Stability of the closed shell nuclei, clearly produces all magic numbers. 2,8,20,50,82....
- 2. spins and parity of nuclear ground state, neutron and proton levels fill independently and following the rules of angular momenta and parity.
- i) even –even nuclei have total ground state angular momentum  $l=0$ .

# PREDICTIONS conti....

- ii) with an odd number of nucleons, nucleons pair off as far as possible so that the resulting angular momentum and spin direction are just that of the single odd particle.
- iii) An odd - odd nucleus will have a total angular momentum which is the vector sum of the odd neutron and odd proton J values. The parity will be the product of proton and neutron parity (condition of being equal or its a state).

# PREDICTIONS conti....

- 3. Magnetic moments of nuclei

In odd nucleus, the total angular momentum  $J$  of the nucleus is equal to the angular momentum  $L$  of the last unpaired nucleons. the magnetic moment of nucleus is provided by odd nucleon only.

- 4. The quadrupole moment of odd  $A$  and odd  $Z$  nuclei is due to unpaired proton. ex at proton numbers 2, 8, 20, 50, 80... (its a quantity of nuclear charges for the state which has maximum component of spin  $I$  in the  $Z$  direction, average of all protons taken)

# PREDICTIONS conti....

- 5. Nuclear Isomerism-Isomers are nuclei in excited energy states and having long half lives ( $>1s$ )
- 6. Beta decay- relative life time can be understood in terms of relative parity and angular momenta.
- 7. Spins and parities of unstable nuclei can be predicted at the time of beta emission.
- 8. Stripping reactions can be explained by this.
- 9. It also explains stability, spin magnetic moment, emission of gamma rays.



# Drawbacks

- 1. magnitude of quadrupole moment deviates from spherical shape.
- 2. Large values of moment far from the spherical shape.
- 3. it fails to explain excited states even-even nuclei.
- 4. it also fails to explain the ground state odd nuclei in the range  $150 \leq A \leq 190$  and at  $A \leq 220$ .
- 5. It is failed to explain the probability of relative transitions.

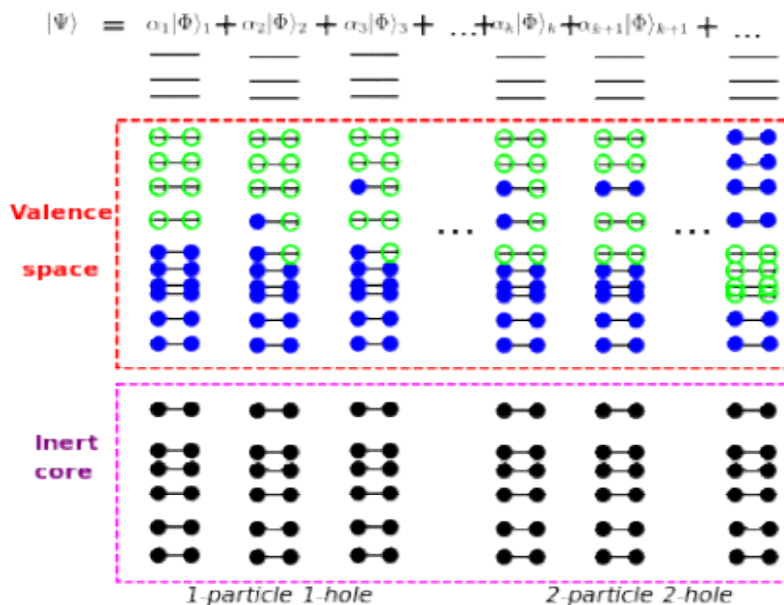
# residual interactions

Residual interactions among valance nucleons are included by diagonalising an effective Hamiltonian in a valance space outside an inert core. As indicated, only single-particle states lying in the valance space are active in the basis used.

For nuclei having two or more valance nucleons (i.e. nucleons outside a closed shell) a residual two-body interaction must be added. This residual term comes from the part of the inter-nucleon interaction not included in the approximative average potential. Through this inclusion different shell configurations are mixed and the energy degeneracy of states corresponding to the same configuration is broken.

These residual interactions are incorporated through shell model calculations in a truncated model space (or valance space). This space is spanned by a basis of many-particle states where only single-particle states in the model space are active. The Schrödinger equation is solved in this basis, using an effective Hamiltonian specifically suited for the model space. This Hamiltonian is different from the one of free nucleons as it among other things has to compensate for excluded configurations.

One can do away with the average potential approximation entirely by extending the model space to the previously inert core and treat all single-particle states up to the model space truncation as active.



## Nuclear Potential and the Shell Model

The [shell model](#) of the nucleus presumes that a given nucleon moves in an effective attractive potential formed by all the other nucleons. If that is true, then the potential is probably roughly proportional to the [nuclear density](#) and therefore could be expressed in the form

$$V = \frac{-V_0}{1 + \exp\left(\frac{r-R}{a}\right)}$$

The parameters in this model of the potential have been evaluated to be approximately:

$$V_0 \approx 57 \text{ MeV} + \text{corrections}$$

$$R \approx 1.25A^{1/3} \text{ fermi}$$

$$a \approx 0.65 \text{ fermi}$$

Note that the radius above is larger than that given by the [nuclear radius formula](#) since it is related to the nuclear force which extends beyond the radius. Two other corrections are typically applied to more nearly fit observations. The first is called the symmetry energy, arising when there is an unequal number of protons and neutrons. Empirically, it is evaluated as

$$\Delta V_s = \pm 27 \text{ MeV} \left[ \frac{N-Z}{A} \right] \begin{array}{l} - \text{ neutrons} \\ + \text{ protons} \end{array}$$

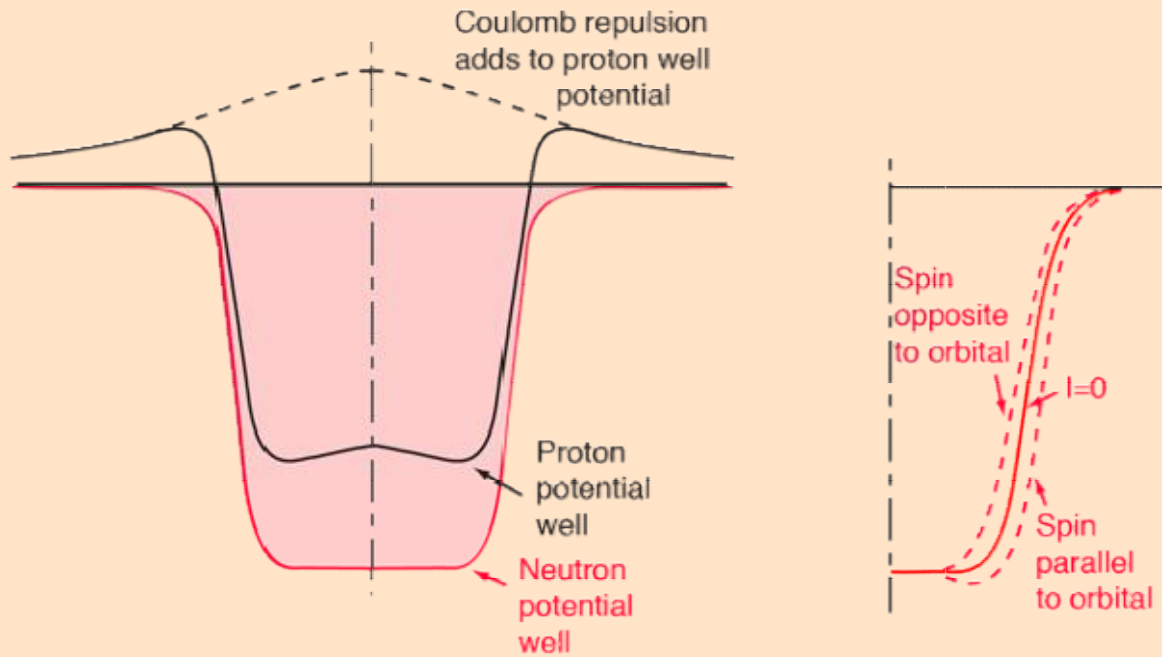
The other correction for protons is the electrostatic repulsion energy, which takes the form

$$V(r) = \frac{Zke^2}{R_c} \left( 1 + \frac{1}{2} \left[ 1 - \left( \frac{r}{R_c} \right)^2 \right] \right) \quad r < R_c$$

$$V(r) = \frac{Zke^2}{r} \quad r > R_c$$

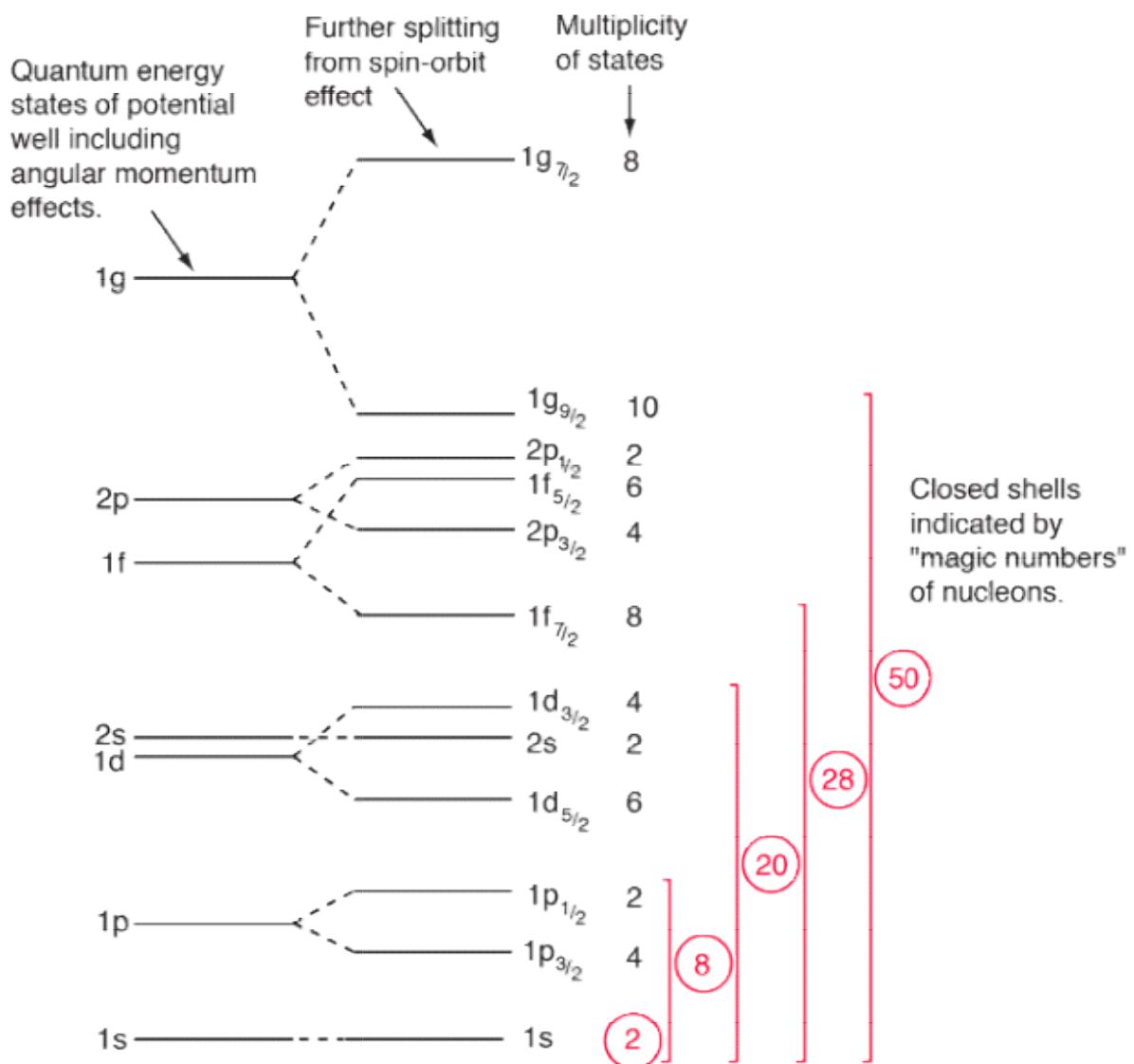
$R_c$  = charge radius, distinct from  $R$ , the model radius for the nuclear potential.

The approximate potentials for neutrons and protons take the general form shown at left below.



The additional correction which must be made is the spin-orbit interaction. Its general effect on the potential well is shown in the sketch above right. If the spin is opposite to the orbital angular momentum, the effective potential well is narrower, giving higher energy in the same manner as the [square well potential](#). This can be seen in the spin-orbit splittings in the [shell model](#) level diagram.

The evidence for a kind of shell structure and a limited number of allowed energy states suggests that a nucleon moves in some kind of effective potential well created by the forces of all the other nucleons. This leads to energy quantization in a manner similar to the [square well](#) and [harmonic oscillator](#) potentials. Since the details of the well determine the energies, much effort has gone into construction of potential wells for the modeling of the observed nuclear energy levels. Solving for the energies from such potentials gives a series of energy levels like that at left below. The labels on the levels are somewhat different from the corresponding symbols for atomic energy levels. The energy levels increase with orbital angular momentum quantum number  $l$ , and the s,p,d,f... symbols are used for  $l=0,1,2,3...$  just like the atomic case. But there is really no physical analog to the principal quantum number  $n$ , so the numbers associated with the level just start at  $n=1$  for the lowest level associated with a given orbital quantum number, giving such symbols as 1g which could not occur in the atomic labeling scheme. The quantum number for orbital angular momentum is not limited to  $n$  as in the atomic case.





# Nuclear Force

- There are **four** basic forces in nature:
- **Gravitational** Force
- **Electromagnetic** Force
- **Strong Nuclear** Force
- **Weak Nuclear** Force
- The **strong nuclear force** is what keeps the **nucleons together** despite having a **similar charge**

- Nucleus is made of fundamental particles known as **protons and neutrons**.
- Enormous **repulsive force** exists among **protons**.
- **Nucleons** are formed with **quarks** which keeps **nucleons** together.
- This **strong force** is **residual colour force**.



- The basic exchange particle is called gluon which works as mediator forces between quarks.
- Both the particles; gluons and quarks are present in protons and neutrons.
- The range of force between particles is not determined by the mass of particles.

- Thus, the force which balanced the repulsion force between the positively charged particles protons are is referred as nuclear attraction which overcomes the electric repulsion force in a short range of order.
- It is **quite stronger** than the **Columbic force** of atomic nuclei and **short range force** for larger nuclei. examples of **weak** and **strong** range nuclear forces.

- **Nuclear Force** is defined as the force exerted between numbers of nucleons. This force is **attractive in nature** which binds **protons and neutrons** in the nucleus together.
- Since the protons are of same positive charge they exert a repulsive force among them. This can result in bursting of the nucleus. Hence to hold them together nuclear force is responsible.

- Because of this attractive **Nuclear Force**, the total mass of the nucleus is less than the summation of masses of nucleons that is protons and neutrons.
- This force is highly attractive between nucleons at a distance of  $10^{-15}$  m or 1 femtometer (fm) approximately from their centers.

- Nuclear forces are of two types namely,
- Strong Nuclear Force
- Weak Nuclear Force.
- Nuclear forces are independent of the charge of protons and neutrons. This property of nuclear force is called charge independence.
- . It depends on the spins of the nucleons .

- Nuclear force is attractive in nature in general cases but it becomes repulsive at very small distances less than 0.7 femtometer (fm).
- Strong nuclear force
- Strong nuclear force is about 100 times stronger than electromagnetism.
- Strong nuclear forces can be applied in two aspects

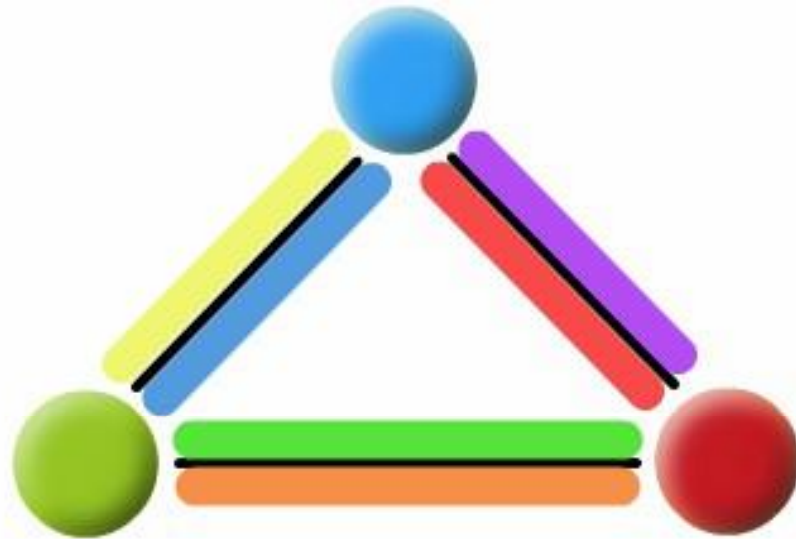
- 1. On larger scale of about 1 to 3 fm distance,  
it is the force that binds the nucleons together  
to form nuclei.
- 2. On smaller scale of about less than 0.8  
fm distance, it is the force that binds quarks  
together to form nucleons that is protons and  
neutrons and also other particles like hadrons.

- Strong interactions bring into account the concept of Colour charge.
- Colour charge is completely different from the visual sense of colour. Color charge is similar to electric charge.
- electromagnetic force is due to electric charge, in the similar way strong nuclear forces are due to colour charge.



- Other particles which do not have colour charge are not responsible for strong forces.
- Fundamentally quarks are coloured charges which feel strong forces.
- Nucleons (protons and neutrons) are considered to be a part of baryon class that contains 3 types of quarks

- each having a colour charge among 3 fundamental colours Red, Blue and Green.
- They are decided according to the rule of Quantum Chromo Dynamics. (a quantum field theory in which the strong interaction is described in terms of an interaction between quarks mediated by gluons, both quarks and gluons being assigned a quantum number called 'colour')



- |   |                              |   |                         |
|---|------------------------------|---|-------------------------|
|  | <b>Blue<br/>Quark/Gluon</b>  |  | <b>Anti-blue Gluon</b>  |
|  | <b>Green<br/>Quark/Gluon</b> |  | <b>Anti-Green Gluon</b> |
|  | <b>Red<br/>Quark/Gluon</b>   |  | <b>Anti-red Gluon</b>   |

- To carry strong nuclear force between quarks or anti quarks, gluons act as mediators.
- Gluons in turn carry colour charge for interaction between quarks and gluons.
- Strong force acts directly on quarks and gluons only.
- These particles interact with each other through strong force.

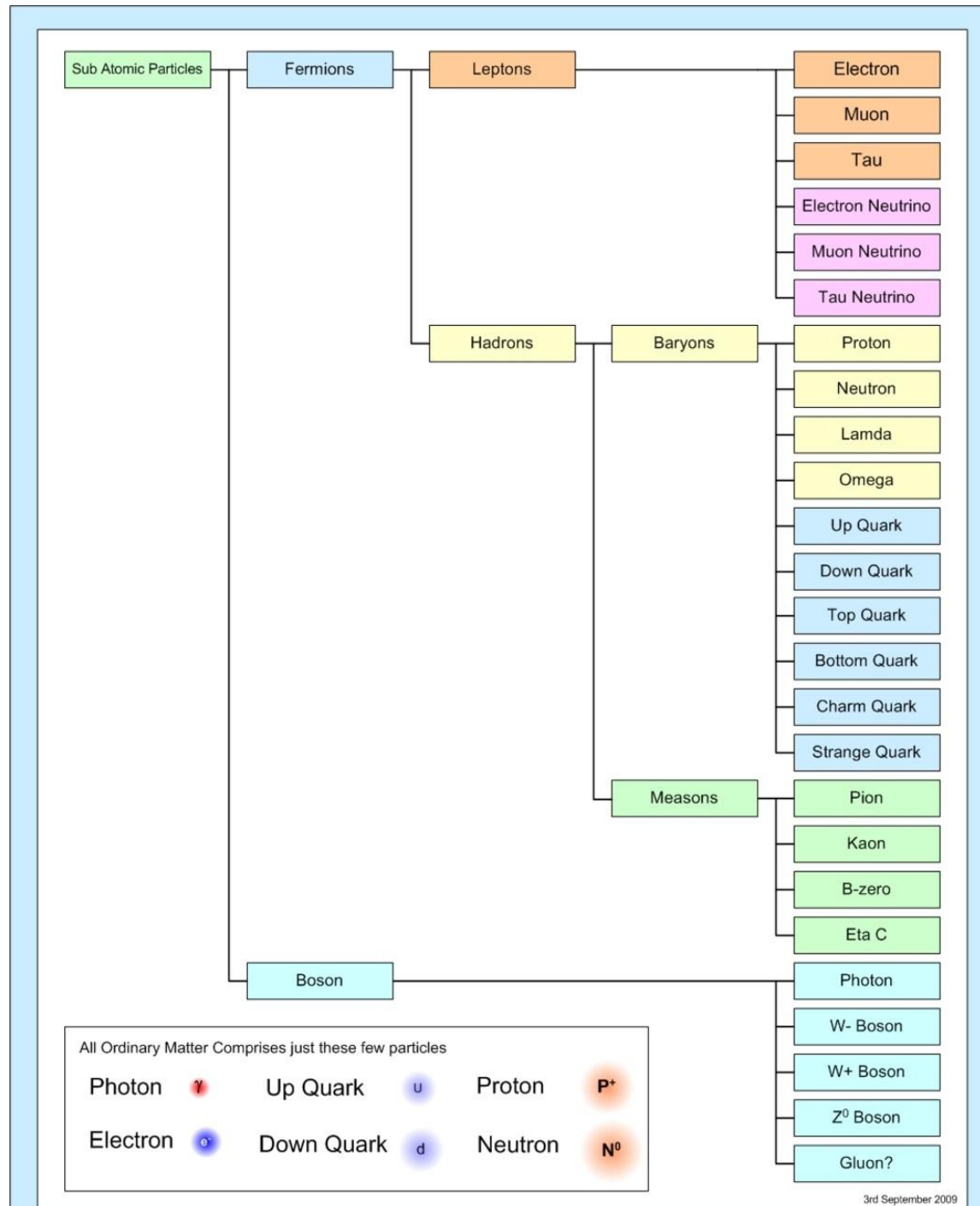
# Examples

- Strong nuclear forces help in holding sub atomic particles of protons together and also the nucleons together at larger scale.
- 1.Strong nuclear force leads to release of energy when heat is generated in Nuclear Power Plant to generate steam for generating electricity.
- Energy is released when a Nuclear Weapon detonates(blow up) which is due to strong nuclear forces.

# Weak Nuclear Force

- Weak Nuclear Force is one of the **four** fundamental force.
- Weak Nuclear Force is caused by the emission or exchange of W and Z bosons
- Weak nuclear forces are very short range because of the heaviness of the **W** and **Z** particles.

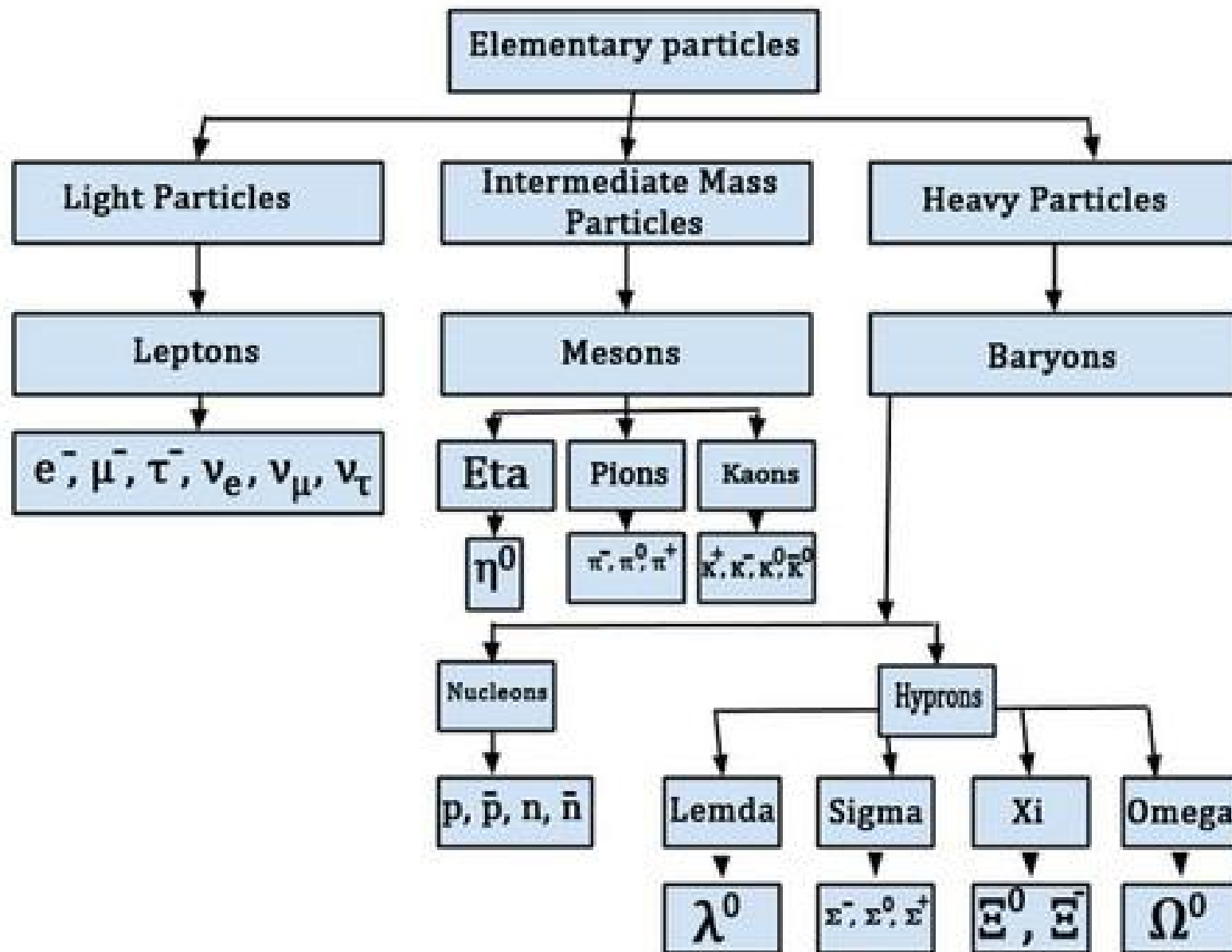
- Weak nuclear force results in the change of one type of quark to another type. This is also known as change of flavor / flavor change of quarks. (There are six types, known as **flavors**, of quarks: **up, down, strange, charm, bottom, and top.**)  
Weak Nuclear Force can transform a neutron into proton or proton into neutron.
- Weak nuclear forces act between quarks and leptons both. (a lepton particle, such as an electron, muon, or neutrino, which does not take part in the strong interaction.)

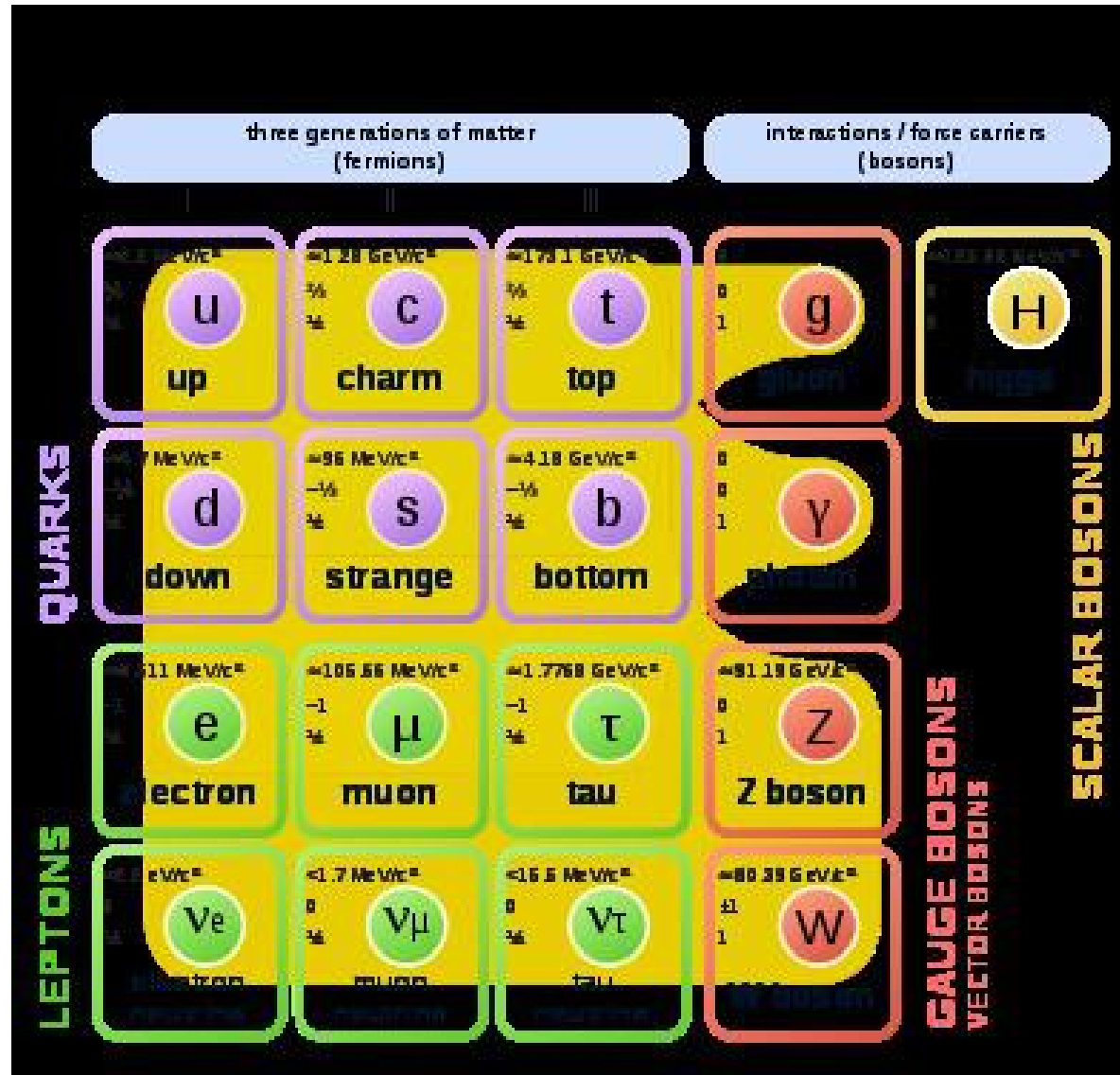


All Ordinary Matter Comprises just these few particles

Photon	$\gamma$	Up Quark	u	Proton	$P^+$
Electron	$e^-$	Down Quark	d	Neutron	$N^0$







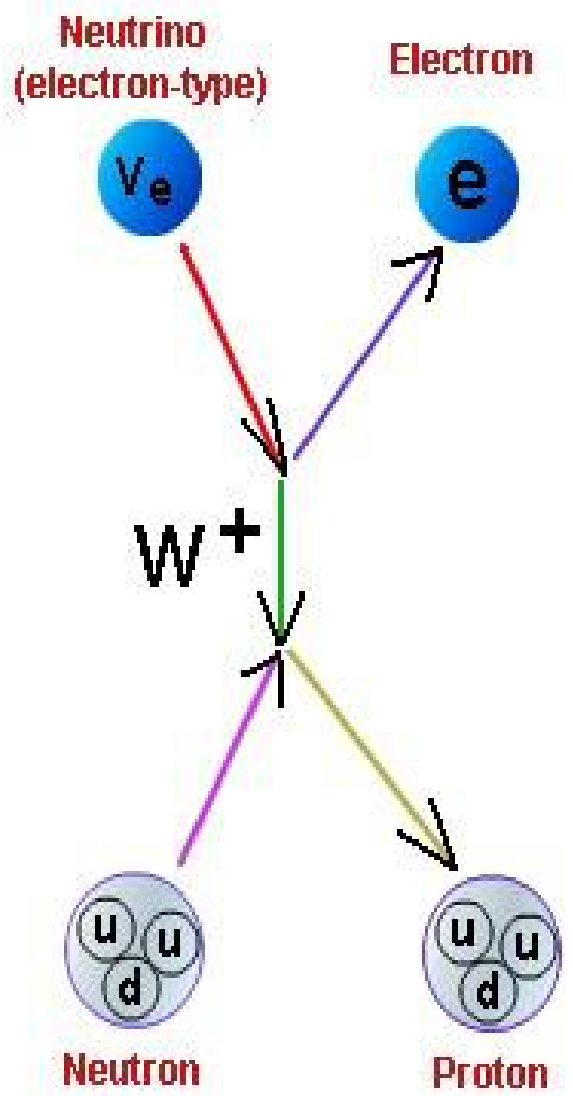
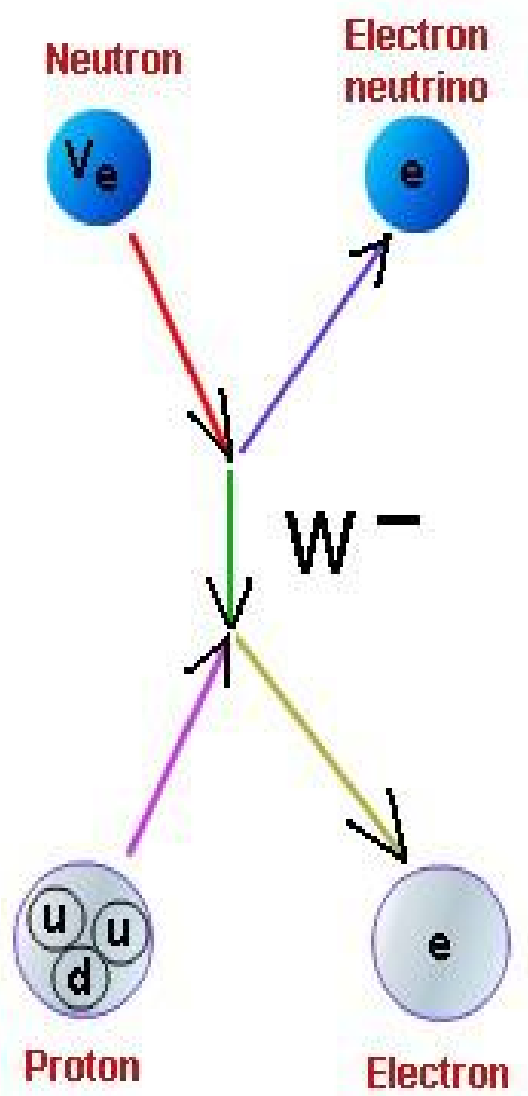
up, down, charm, strange, top and bottom.



up, down, charm, strange, top and bottom.

- **Weak interaction** is responsible for the **flavor change of quarks and leptons.**
- **The significance of weak nuclear force in flavor change of quarks** makes it the interaction indulged in many **decay phenomenons** of nuclear particles which need **a change of quark from one type to another.**

- Weak nuclear force is of two types:
- 1.Charged current Nuclear Force and
- 2.Neutral current Nuclear Force.
  
- Charged current nuclear force is so called  
because this force is carried by electric charge  
carriers i.e. **W+** and **W-** boson particles.



- These forces occurs in many reactions namely,
- Radioactive decay
- Beta decay
- Burning of sun
- Initiating the process of hydrogen fusion in stars.
- In production of deuterium
- Radiocarbon dating and
- Radio luminescence.

- **Weak Nuclear Force Examples**

- 

- Weak Nuclear Force is involved in many phenomenon in nature:



## Nuclear Force

There are four basic forces in nature:

1. Gravitational Force
2. Electromagnetic Force
3. Strong Nuclear Force
4. Weak Nuclear Force

The strong nuclear force is what keeps the nucleons together despite having a similar charge

The nucleus is held by the forces which protect them from the enormous repulsion forces of the positive protons. It is a force with short range and not similar to the electromagnetic force. We know that the nucleus is made up with its fundamental particles that are the protons and neutrons. These are formed with quarks which are held together with strong force. This strong force is residual colour force. The basic exchange particle is called gluon which works as mediator forces between quarks. Both the particles; gluons and quarks are present in protons and neutrons.

The range of force between particles is not determined by the mass of particles. Thus, the force which balanced the repulsion force between the positively charged particles protons are is referred as nuclear attraction which overcomes the electric repulsion force in a short range of order. It is quite stronger than the Columbic force of atomic nuclei and short range force for larger nuclei. examples of weak and strong range nuclear forces.

Nuclear Force is defined as the force exerted between numbers of nucleons. This force is attractive in nature which binds protons and neutrons in the nucleus together. Since the protons are of same positive charge they exert a repulsive force among them. This can result in bursting of the nucleus. Hence to hold them together nuclear force is responsible.

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Nuclear forces are of two types namely,

1. Strong Nuclear Force
2. Weak Nuclear Force.

Nuclear forces are independent of the charge of protons and neutrons. This property of nuclear force is called charge independence. It depends on the spins of the nucleons that is whether they are parallel or no and also on the non central or tensor component of nucleons.

Nuclear force is attractive in nature in general cases cut it becomes repulsive at very small distances less than 0.7 femtometer (fm).

### Strong Nuclear Force

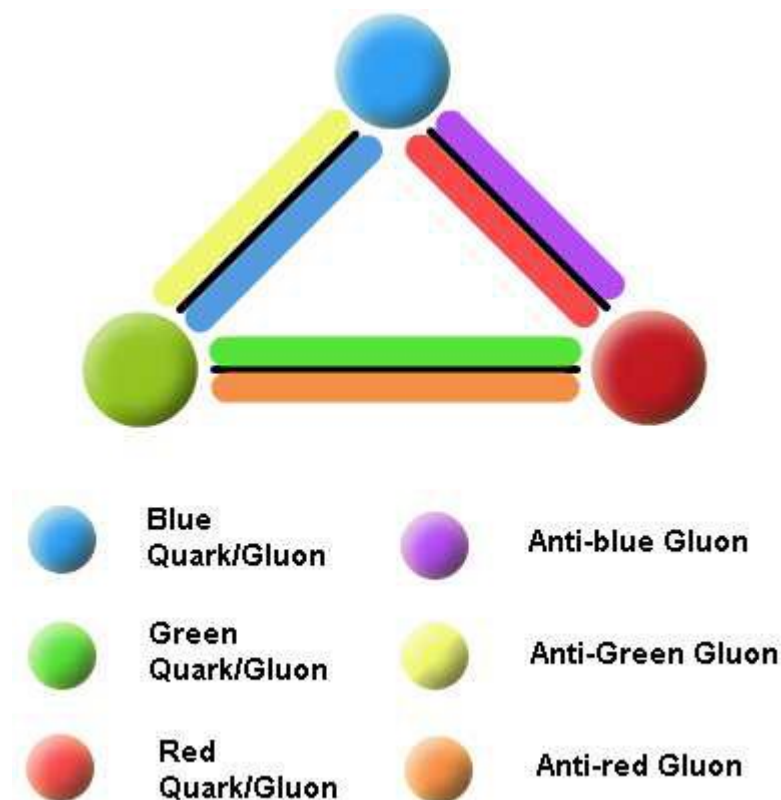
Strong nuclear force is about 100 times stronger than electromagnetism. These forces is also known as strong interactions.

Strong nuclear forces can be applied in two aspects: One is on Larger Scale and other on Lower Scale.

1. On larger scale of about 1 to 3 fm distance, it is the force that binds the nucleons together to form nuclei.
2. On smaller scale of abut less than 0.8 fm distance, it is the force that binds quarks together to form nucleons that is protons and neutrons and also other particles like hadrons.

Strong interactions bring into account the concept of Colour charge. It is completely different from the visual sense of colour. Color charge is similar to electric charge. As the electromagnetic force is due to electric charge, in the similar way strong nuclear forces are due to colour charge. Other particles which do not have colour charge are not responsible for strong forces. Fundamentally quarks are colour charges which feel strong forces.

Nucleons (protons and neutrons) are considered to be a part of baryon class that contains 3 types of quarks, each having a colour charge among 3 fundamental colours Red, Blue and Green. They are decided according to the rule of Quantum Chromo Dynamics.



To carry strong nuclear force between quarks or anti quarks, gluons act as mediators. Gluons in turn carry colour charge for interaction between quarks and gluons. Strong force acts directly on quarks and gluons only. These particles interact with each other through strong force.

Examples

Strong nuclear forces help in holding sub atomic particles of protons together and also the nucleons together at larger scale.

1. Strong nuclear force leads to release of energy when heat is generated in Nuclear Power Plant to generate steam for generating electricity.
2. Energy is released when a Nuclear Weapon detonates which is due to strong nuclear forces.

## Weak Nuclear Force

Weak Nuclear Force is one of the four fundamental force. Electromagnetic force, gravitational force and strong nuclear force are the other forces.

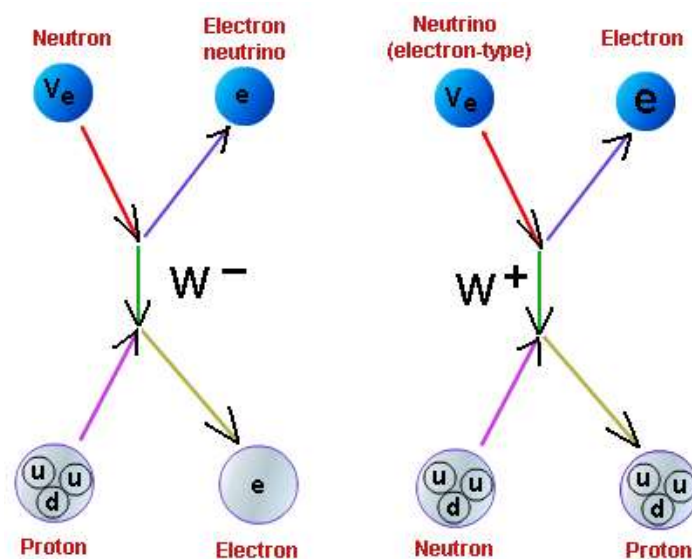
Weak Nuclear Force is caused by the emission or exchange of W and Z bosons. Weak nuclear forces are very short range because of the heaviness of the W and Z particles. Weak nuclear force results in the change of one type of quark to another type. This is also known as change of flavor / flavor change of quarks.

Weak Nuclear Force can transform a neutron into proton or proton into neutron. Weak nuclear forces act between quarks and leptons both. Weak interaction is responsible for the flavor change of quarks and leptons. The significance of weak nuclear force in flavor change of quarks makes it the interaction indulged in many decay phenomenons of nuclear particles which need a change of quark from one type to another.

Weak nuclear force is of two types:

1. Charged current Nuclear Force and
2. Neutral current Nuclear Force.

Charged current nuclear force is so called because this force is carried by electric charge carriers i.e.  $W^+$  and  $W^-$  boson particles.



Neutral current nuclear force is carried by neutral particles i.e. Z boson particles.

**These forces occurs in many reactions namely,**

1. Radioactive decay
2. Beta decay
3. Burning of sun
4. Initiating the process of hydrogen fusion in stars.
5. In production of deuterium
6. Radiocarbon dating and
7. Radio luminescence.

**Weak Nuclear Force Examples**

Weak Nuclear Force is involved in many phenomenon in nature:

**Charged Current Nuclear Force:**

- A charged electron or muon (lepton) with -1 charge absorbs  $W^+$  particle of charge +1 and is converted into neutrino of charge zero. Type of neutrino will be the same as that of lepton.
- A down quark can be converted into up type quark by releasing  $W^-$  particle.
- An up type quark is converted into down type by absorbing  $W^-$  particle or emitting  $W^+$  boson particle.
- A  $W$  boson particle is not stable. Therefore it will decompose or decay in a short time.

**Current Nuclear Particle:**

A quark or lepton can emit or absorb  $Z$  boson particle of zero charge.  $Z$  boson is also unstable hence decomposes very soon.

It can transform a neutron into proton and a proton into neutron.

Neutron ( $n$ )  $\rightarrow$  proton ( $p$ ) + electron ( $e^-$ ) + anti neutrino ( $\bar{\nu}$ ) - - - -> **Electron decay**

Proton ( $p$ )  $\rightarrow$  neutron ( $n$ ) + positron ( $e^+$ ) + neutrino ( $\nu$ ) - - - -> **Positron decay.**

It is responsible for radio active decay of particles. It initiates the process of hydrogen fusion at stars. It is also responsible for beta decay which is a form of radioactivity.

It is also responsible for production of deuterium and helium from hydrogen which helps in burning of sun and powers the sun's thermonuclear energy. It is also possible to perform radio carbon dating with weak nuclear force since

**C – 14** decays into **N – 14** due to weak nuclear force. Weak nuclear force can also create radio luminescence. It also helps in the formation of heavy nucleus.