Laws of vibrating of String :-

(i) Law of length :

For a given string under a given tension, the fundamental frequency varies inversely as length of vibrating string.

$$\text{ie, } f \propto \frac{1}{l}$$

(ii) Law of tension :

For a given vibrating length and material of a string, the fundamental frequency varies directly as the square root of the tension.

$$\text{ie, } f \propto \sqrt{T}$$

(iii) Law of mass :

For a given tension and a given vibrating length of the string, the fundamental frequency varies inversely as the square root of the mass per unit length of the string

$$\text{ie, } f \propto \frac{1}{\sqrt{m}}$$

Phase velocity (or) Wave velocity :

The velocity with which a plane progressive wave travels in the forward direction is known as phase velocity.

(or)

The velocity with which a wave of single frequency (or) wave length travels through a medium is known as wave velocity.

plane wave moving in the +ve x-direction is given by

$$y = a \sin(\omega t - kx)$$

here $\omega = 2\pi f$ is the angular frequency.

[illegible]

$k = \frac{2\pi}{\lambda}$ is the propagation constant of the wave.

then $\frac{\omega}{k} = \frac{2\pi f}{(2\pi/\lambda)} = f\lambda$

But $v = f\lambda$

$$\frac{\omega}{k} = v$$

and $(\omega t - kx)$ is the phase of the wave motion and is a const.

i.e. $\omega t - kx$ = constant.

diff. w.r.t 't'

$$\omega - k\frac{dx}{dt} = 0$$

$$\omega = k\frac{dx}{dt}$$

$$\therefore \frac{dx}{dt} = v_p = \frac{\omega}{k}$$

which is the wave velocity (or) phase velocity.

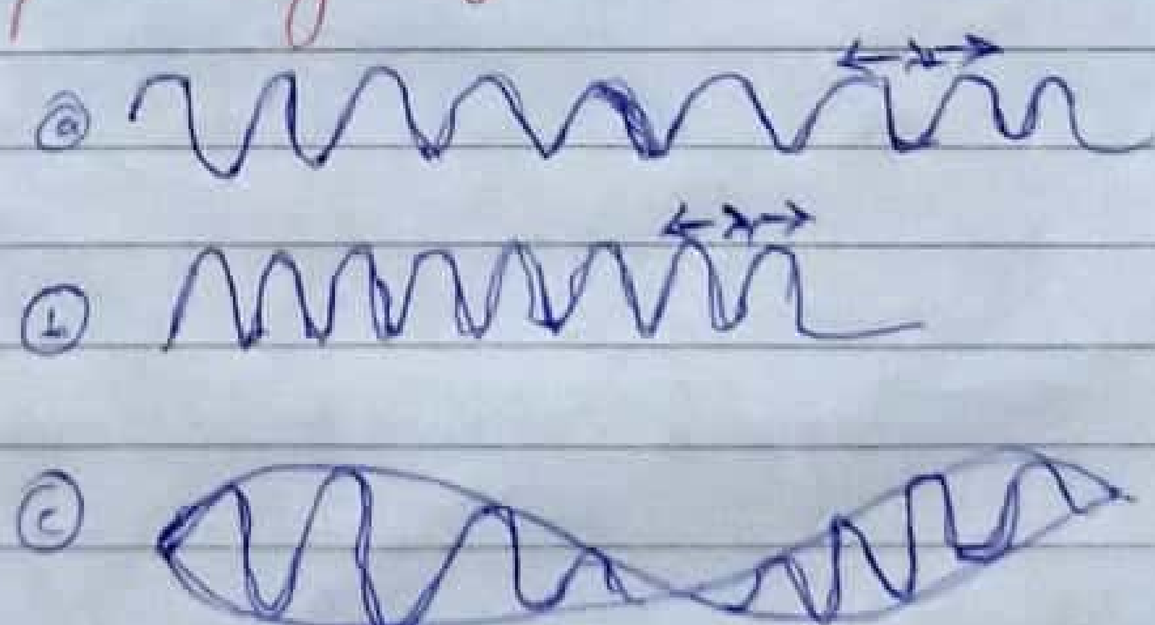
* Group Velocity:

The velocity with which the wave envelop moves (or) maximum amplitude of the group of the waves moves in a medium is known as group velocity.

(or) The velocity with which the energy in the group of waves is transmitted is known as group velocity.

* Expression for group velocity (v_g):

Consider a wave group consisting of two waves of equal amplitude but slightly different frequencies is as shown in the fig.



Two waves of same amplitudes but slightly diff frequency & corresponding resultant wave.

(2)

Their displacement are given by $y_1 = a\sin(\omega_1 t - k_1 x)$
$y_2 = a\sin(\omega_2 t - k_2 x)$

where $\omega_1 = \omega + \Delta\omega$ and $\omega_2 = \omega - \Delta\omega$ are angular frequency, $k_1 = k + \Delta k$ and $k_2 = k - \Delta k$ are the propagation const. From the principle of superposition, the resultant wave is given by

$$y = y_1 + y_2$$
$$y = a\sin(\omega_1 t - k_1 x) + a\sin(\omega_2 t - k_2 x)$$
$$y = a[\sin(\omega_1 t - k_1 x) + \sin(\omega_2 t - k_2 x)]$$

Using the relation, $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

$$y = 2a\sin\left[\left(\frac{\omega_1+\omega_2}{2}\right)t - \left(\frac{k_1+k_2}{2}\right)x\right] \times$$
$$\cos\left[\left(\frac{\omega_1-\omega_2}{2}\right)t - \left(\frac{k_1-k_2}{2}\right)x\right]$$

Using the value of ω_1, ω_2, k_1 & k_2

we get -

$$y = 2a\sin(\omega t - kx)\cos(\Delta\omega t - \Delta kx)$$
$$y = 2a\cos(\Delta\omega t - \Delta kx)\sin(\omega t - kx)$$

put $2a\cos(\Delta\omega t - \Delta kx) = R$

$$\therefore \quad y = R\sin(\omega t - kx)$$

This equation shows that the resultant wave is a sinusoidal wave of angular frequency to amplitude R is as shown in the fig.

The amplitude R of the resultant wave propagates itself as a wave with a phase velocity $\left(\frac{\Delta\omega}{\Delta k}\right)$.

Since the phase velocity of the resultant envelope of the group is called group velocity

$$\therefore v_g = \frac{\Delta\omega}{\Delta k}$$

For smaller variation in frequency $(\Delta\omega)$ & propagation constant (Δk) the group velocity is given by $\therefore v_g = \lim_{\Delta \to 0} \frac{\Delta\omega}{\Delta k} = \frac{d\omega}{dk}$

$$\therefore \boxed{v_g = d\omega/dk}$$

Hrayizan

Relation between group velocity and phase velocity:

Consider the equations for phase velocity & group velocity

$v_p = \frac{\omega}{k}$ and $v_g = \frac{d\omega}{dk}$

Diff. the equation v_p wrt 'k'

$$\frac{dv_p}{dk} = \frac{1}{k}\left(\frac{d\omega}{dk}\right) + \omega\left(-1/k^2\right)$$

$$= \frac{1}{k}\left(d\omega/dk\right) - \left(\omega/k^2\right)$$

$$k\frac{dv_p}{dk} = \frac{d\omega}{dk} - \frac{\omega}{k}$$

$$\frac{d\omega}{dk} = \frac{\omega}{k} + k\frac{dv_p}{dk}$$

(or) $$v_g = v_p + k\frac{dv_p}{dk} \longrightarrow (1)$$

But $k = \frac{2\pi}{\lambda}$ $\therefore \frac{dk}{d\lambda} = \frac{-2\pi}{\lambda^2}$

$$v_g = v_p + k\frac{dv_p}{dk} \cdot \frac{d\lambda}{dk}$$

Substituting for k and $\frac{dk}{d\lambda}$ values

we get - $v_g = v_p + \frac{2\pi}{\lambda}\left(\frac{dv_p}{dk}\right)\left(\frac{-\lambda^2}{2\pi}\right)$

$$\boxed{v_g = v_p - \lambda\left(dv_p/dk\right)}$$

This is the relation b/w group velocity v_g & wave velocity v_p in a dispersive medium.

Acoustics of Buildings

Acoustics of Buildings is the branch of physics and engineering in which deals with the generation and propagation and reception of sound.

* **Reverberation**

The persistance of audible sound in a hall or auditorium even after the source of sound stop emitting sound waves is called reverberation.

* **Reverberation time:**

The time during which the audible sound persists in the hall or auditorium even after the source of sound stop emitting sound waves is called reverberation time.

Reverberation time depends on the following factors.

* The nature of reflecting materials on the walls, ceiling roof, floor. etc
* The frequency of sound emitting in the hall.
* The co-efficient of absorption of materials, present in the hall.
* The volume of the hall
* The intensity of sound wave emitted in the hall.

Absorption co-efficient:

The ratio of sound energy absorbed by a material body to the total energy incident on the material body is known as absorption co-efficient.

** Sabine's formula for Reverberation time.

Consider an auditorium or a hall in which sound waves are produced and get reflected from the surface of the walls. During the propogation the sound travels an average distance between two successive reflections, the average distance is called mean free path. It is given by

$$d = \frac{4V}{S}$$

where,

V - volume of the hall
S - Surface area of the hall

Then, the time taken between the two successive reflections is given by.

$$t_1 = \frac{\text{distance between two successive collisions}}{\text{velocity of sound}}$$

$$t_1 = \frac{d}{v} = \frac{[4V/S]}{v} = \frac{4V}{Sv}$$

The average number of reflections of sound in a

$$n = \frac{t}{t_1} = \frac{t}{[4V/Sv]} = \frac{Svt}{4V}$$

Let a_f be the fraction of sound absorbed at a single absorption, then the fraction of sound reflected from the wall will be $(1-a_f)$

After two successive reflections, the fraction of sound reflected from the walls will be $(1-a_f)^2$.

Similarly, after n reflections, the fraction sound reflected from the walls will be $(1-a_f)^n$.

Let I_0 ~~be it~~ & I_t be the initial & final intensity of sound respectively then they are related as

$I_t = I_0 \times$ fraction of sound reflected from the walls after n reflections.

$$I_t = I_0 (1-a_f)^n$$

But $$n = \frac{Svt}{4V}$$

$$I_t = I_0 (1-a_f)^{\frac{Svt}{4V}}$$

For reverberation time, $t = T$

$$I_t = I_0 (1-a_f)^{\frac{SvT}{4V}} \quad \text{------>} \; (1)$$

By the definition of reverberation time.

$$I_t = I_0 \times 10^{-6} \quad \text{--->} \; (2)$$

By comparing (1) & (2), we get

$$I_0 (1-a_f)^{\frac{SvT}{4V}} = I_0 \times 10^{-6}$$

$$10^{-6} = (1-a_f)^{\frac{SvT}{4V}}$$

Take log on both side.

$$\log_e(10^{-6}) = \log_e(1-a_f)^{\frac{S v T}{4V}}$$

$$= \frac{S v T}{4V} \log_e(1-a_f) \qquad [\because \log a^n = n \log a]$$

$$T = \frac{4V \log_e(10^{-6})}{S v \log(1-a_f)}$$

$$T = \frac{2.303 \times 4V \log_{10}(10^{-6})}{S v \log(1-a_f)}$$

But $\log(1-a_f) = -a_f$

$$T = \frac{-55.272V}{S v(-a_f)} = \frac{55.272V}{S v a_f}$$

But velocity of sound at room temp^r $v = 350\, ms^{-1}$

$$T = \frac{55.272V}{350 a_f S}$$

$$T = \frac{0.1579V}{a_f S'} \longrightarrow (3)$$

But fraction of sound absorbed by the materials at single absorption is given by

$$a_f = \frac{a_1 S_1 + a_2 S_2 + a_3 S_3 + \cdots a_n S_n}{S_1 + S_2 + S_3 + \cdots S_n}$$

where $a_1; a_2, a_3, \cdots$ are the absorption co-efficients of the materials in the hall and $S_1, S_2, S_3 \ldots S_n$ are the surface areas exposed to sound waves respectively.

Above equation can be written as

$$a_f = \frac{a_1 S_1 + a_2 S_2 + a_3 S_3 + \cdots a_n S_n}{S_1 + S_2 + S_3 + \cdots S_n} = \frac{\Sigma a S}{S} \longrightarrow (4)$$

Substitute equation (4) in equation (3), we get

$$T = \frac{0.1579V}{\left[\frac{\Sigma a S}{S'}\right] \times S'}$$

$$\boxed{T = \frac{0.1579\,V}{\Sigma a s}}$$

This equation is called Sabine's formula for reverberation time.

Above equation indicates that the reverberation time is

* Directly proportional to volume of the hall
* And inversly proportional to the surface area & absorption co-efficient of the hall.

If velocity of sound in air $= 330\,ms^{-1}$

Then

$$\boxed{T = \frac{0.165\,V}{\Sigma a s}}$$

Acoustic aspects of Hall (or) Auditorium.

* **Adequate loudness and uniform distribution of sound.**

For an acoustic hall the sound must be loud, so it can be increased by parabolic reflector walls.

* **Absence of ecos**

For an acoustically good halls, there should not be any echo. The echo's are mainly due to reflection of sound from walls & ceiling and that can be minimised by using suitable sound absorbing materials.

* **Elimination of extraneous noise**

The extraneous noise can be minimised by using double (or) tripple doors and windows each with its own frame work.

* **Absence of resonance.**

Resonance can be minimized by constructing the large hall having large surface area.

* **optimum ~~sound~~ reverberation time**

Optimum reverberation time can be produced by providing open window, using heavy curtains with the folds.

G.R Revannasiddappa

(R.S)

4th SEMESTER Notes

the ~~Ferm~~ lowest level two more the next and so on. until all the electrons in the metal have been occupied. The energy of highest occupied level is called the fermi energy level (E_F)

The probability that a particular quantum state having an energy E is occupied at temperature (T) Kelvin is given by

$$F(E) = \frac{1}{e^{\frac{E-E_F}{KT}}+1}$$

where E_F is the fermi energy of the metal.

k - Boltzmann's constant.

If $T = 0°K$ then $f(E) = 1$ for $E < E_F$

$f(E) = 0$ for $E > E_F$.

If $T > 0°K$ $f(E) = \frac{1}{e^0+1} = \frac{1}{1+1} = \frac{1}{2}$

and $E = E_F$.

<u>Expression for Fermi energy at Absolute Zero Kelvin</u>: ($E_{F(0)}$)

Fermi energy at zero Kelvin $E_{F(0)}$ is defined as the energy corresponding to highest energy state occupied by the free electron.

<u>OR</u>

It is defined as the maximum kinetic energy that a free electron can have at zero kelvin. Free electrons in solid obey fermi dirac statistics i.e. only one electron can occupy each quantum state and maximum of two electrons in each energy level. The Probability that a state having energy E filled by an electron in thermal equilibrium at temperature T is given by

$$F(E) = \frac{1}{e^{\frac{E-E_F}{KT}}+1} \quad \text{———} \quad (1)$$

where K - Boltzmann Constant.

T - absolute temperature.

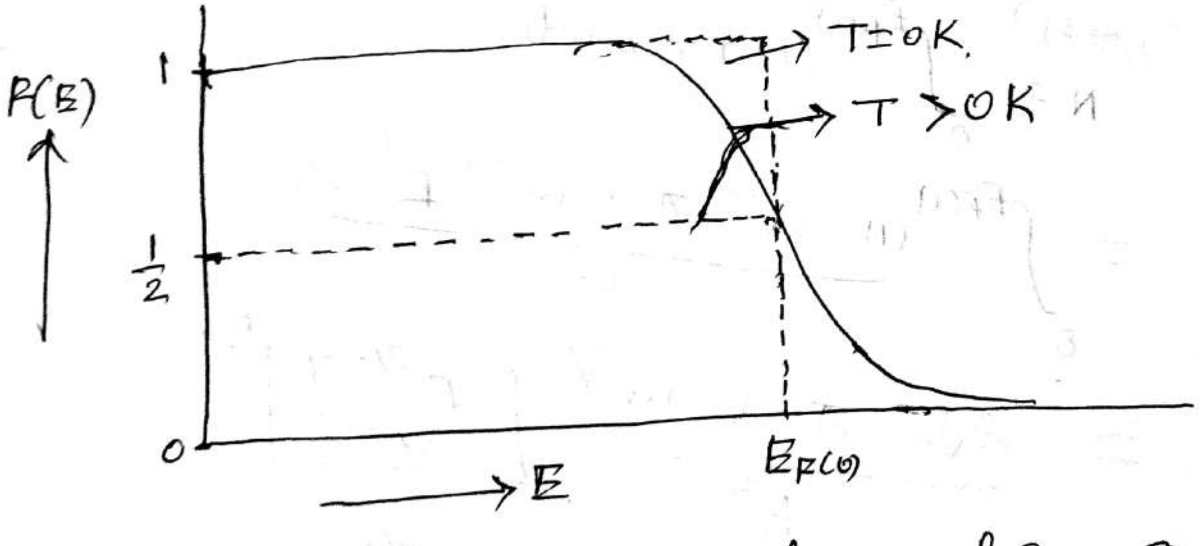
E_F - Fermi energy.

At temperature $T = 0^{\circ}K$ $f(E) = 1$ if $E < E_F$

$f(E) = 0$ if $E > E_F$

At temperature $T > 0^{\circ}K$, and if $E = E_F$

Then $F(E) = \frac{1}{e^{0}+1} = \frac{1}{1+1} = \frac{1}{2}$



Therefore Fermi energy at any temperature greater than Zero Kelvin is defined as energy of that level for which probability of occupation is ½.

F(E) also represents fraction of possible state which are filled by electrons. If these are $g(E)\,dE$ as allowed quantum state in the energy range E and $E+dE$ and if $N(E)$ is the number of electrons in the same range

Then $N(E) = g(E)\, f(E)\, dE$

$$\therefore f(E) = \frac{N(E)}{g(E)\, dE} \quad \text{——— (2)}$$

Electrons in metal can be considered as particles in three dimensional box therefore number of quantum states in the energy range E and E+dE is given by.

$$g(E)\, dE = \frac{8\sqrt{2}\,\pi V\, m^{3/2} E^{1/2}\, dE}{h^3} \quad \text{——— (3)}$$

where $m \rightarrow$ mass of electron.

V – volume of electron gas. i.e. volume of solid crystal. If there are N free electrons in it the highest state filled its energy $E = E_{F(0)}$ at ~~[illegible]~~ Zero kelvin (~~[illegible]~~ K)

(T=0°K)

$$N = \int_0^{E_{F(0)}} f(E)\, g(E)\, dE$$

$$= \int_0^{E_{F(0)}} (1)\, \frac{8\sqrt{2}\,\pi V m^{3/2} E^{1/2}\, dE}{h^3}$$

$$= \frac{8\sqrt{2}\,\pi V m^{3/2}}{h^3} \left[\frac{E^{3/2}}{3/2} \right]_0^{E_{F(0)}}$$

$$= \frac{8\sqrt{2}\,\pi V m^{3/2}}{h^3} \times \frac{2}{3} \times E_{F(0)}^{3/2}$$

$$N = \frac{8(2\sqrt{2})\,\pi V m^{3/2} E_{F(0)}^{3/2}}{3h^3}$$

$$E_{F(0)}^{3/2} = \frac{3Nh^3}{8(2\sqrt{2})\,\pi V m^{3/2}}$$

$$E_{F(0)}^{3/2} = \frac{3Nh^3}{8(2\cdot 2^{1/2})\pi V m^{3/2}}$$

$$E_{F(0)}^{3/2} = \frac{3Nh^3}{8(2^{3/2})\pi V m^{3/2}}$$

$$E_{F(0)}^{3/2} = \frac{3Nh^3}{8\pi V (2m)^{3/2}}$$

$$E_{F(0)}^{3/2} = \frac{3N}{8\pi V}\left[\frac{h^2}{2m}\right]^{3/2}$$

$$\boxed{E_{F(0)} = \left(\frac{3N}{8\pi V}\right)^{2/3}\left[\frac{h^2}{2m}\right]}$$

This is the expression for fermi energy at Absolute zero.

The quantity $\frac{N}{V}$ is the density of free electrons ie; No of free electrons per unit volume.

Expression for Average kinetic energy of Three dimensional electron gas at Zero Kelvin :-

Fermi energy $E_{F(0)}$ at zero kelvin is defined as the energy corresponding to highest state occupied by the free electron.

OR

It is defined as the maximum kinetic energy that a free electron can have at zero kelvin.

Therefore Average K.E at absolute zero Kelvin is

$$\bar{E}_0 = \frac{\text{Total energy of all free electrons}}{\text{Total number of electrons.}}$$

$$= \int_0^{E_{F(0)}} \frac{E\, f(E)\, g(E)\, dE}{N}$$

$$\bar{E}_0 = \frac{\int_0^{E_{F(0)}} \frac{E\,(1)\, 8\sqrt{2}\,\pi V m^{3/2} E^{1/2}\, dE}{h^3}}{\frac{8\,(2\sqrt{2})\,\pi V\, m^{3/2}\, E_{F(0)}^{3/2}}{3h^3}}$$

$$= \frac{\frac{8\sqrt{2}\,\pi V\, m^{3/2}}{h^3}\ \frac{E_{F(0)}^{5/2}}{5/2}}{\frac{8(2\sqrt{2})\pi V m^{3/2}\, E_{F(0)}^{3/2}}{3h^3}}$$

$$\bar{E}_0 = \frac{\frac{8\,(2\sqrt{2})\,\cancel{\pi V m^{3/2}}}{5h^3}\ E_{F(0)}^{5/2}}{\frac{8\ 2\sqrt{2}\ \cancel{\pi V m^{3/2}}}{3h^3}\ E_{F(0)}^{3/2}}$$

$$\bar{E}_0 = \frac{1}{5h^3} \times \frac{3h^3}{1}\ \frac{E_{F(0)}^{5/2}}{E_{F(0)}^{3/2}}$$

$$\boxed{\bar{E}_0 = \frac{3}{5}\, E_{F(0)}}$$

Thus average K.E is 3/5 times of Fermi energy at zero Kelvin.

Applications of Bose – Einstein Statistics

Planck's law of Radiation

Assumptions:

1. Photons do not interact among themselves.
2. photons interact only with the atoms of wall of cavity.
3. Photons are indistinguishable and many photons can have the same energy.
4. photons are considered as bosons and they obey Bose – Einstein statistics.
5. Radiant energy occurs in energy packets or quanta or photons each of energy $E = h\nu$.
6. photons are of zero rest mass and spin quantum number 1.

Derivation: The particles having various energy level for a system obeying Bose-Einstein distribution law is given by

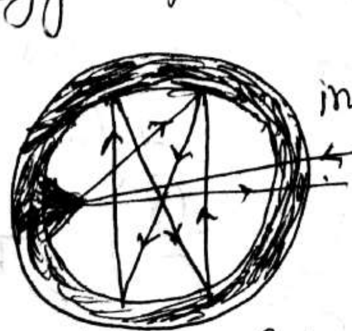
$$n_i = \frac{g_i}{e^{(\alpha+\beta E_i)} - 1} \quad \text{——— (1)}$$

In case of photon gas the constancy of number of particles does not apply i.e. $\sum_i dn_i \neq 0$ i.e. $\alpha = 0$ for a gas of photons and $\beta = \frac{1}{KT}$

Since $\alpha = 0$

we have
$$n_i = \frac{g_i}{e^{\frac{E_i}{KT}} - 1} \quad \text{——— (2)}$$

Since the cavity is large compared to the wavelength of radiation, the energy spectrum of photons is taken to be continuous.



Black body

Momentum Space.

In the limit of very close energy levels, we have

$$n(E)\,dE = \frac{g(E)\,dE}{e^{\frac{E}{KT}} - 1} \qquad \text{——— (3)}$$

Let the volume of the blackbody radiation be V. The number of states in a black body radiation in the frequency range ν and $\nu + d\nu$ can be obtained by calculating the spherical volume bounded by two spheres of radii p and $(p+dp)$

We know that momentum $p = \frac{h\nu}{c}$ ——— (4)

$p = mc \quad \nu\lambda = c$

$p = m\nu\lambda = \frac{h\nu}{c}$ Diff $dp = \frac{h}{c}\,d\nu$ ——— (5)

volume bounded by the sphere $V = 4\pi p^2 dp$

According to uncertainty principle volume of each cell $= h^3$

$\therefore$ number of cells per unit volume of radiation $= \frac{4\pi p^2 dp}{h^3}$

The number of states in the black body radiation b/n p and $p+dp$ in volume V with two polarization states is

$$g(p)\,dp = 2\left(\frac{4\pi p^2 dp\, V}{h^3}\right) \qquad \text{——— (6)}$$

Density of states in the frequency range ν and $\nu + d\nu$ is given by

$$g(\nu)\,d\nu = 2 \times \frac{4\pi}{h^3}\left(\frac{h\nu}{c}\right)^2\left(\frac{h}{c}\,d\nu\right)V$$

$$g(\nu)\,d\nu = \frac{8\pi V}{c^3}\,\nu^2\,d\nu \qquad \text{——— (7)}$$

Substituting in eqn (3) the number of photons in the frequency range ν and $\nu + d\nu$ is given by

$$n(\nu)\,d\nu = \left(\frac{8\pi V \nu^2\, d\nu}{c^3}\right)\frac{1}{\left(e^{h\nu/KT} - 1\right)} \qquad (\because E = h\nu)$$

——— (8)

Since each photon has energy ($E = h\nu$) the energy density $E(\nu)d\nu$ defined as the amount of energy per unit volume lying between the frequency range ν and $\nu + d\nu$ is given by

$$E(\nu)d\nu = \frac{h\nu \cdot n(\nu)d\nu}{V}$$

$$= \cancel{h\nu}$$

$$E(\nu)d\nu = \frac{h\nu}{\cancel{V}} \times \left(\frac{8\pi \cancel{V} \nu^2 d\nu}{c^3}\right) \frac{1}{e^{h\nu/KT} - 1}$$

$$\boxed{\text{Energy density, } E(\nu)d\nu = \frac{8\pi h \nu^3}{c^3} \times \frac{1}{e^{h\nu/KT} - 1} d\nu} \quad (9)$$

This Equation represents Planck's radiation law for blackbody radiation.

In terms of wavelength

We know that $\nu\lambda = c \quad \therefore \nu = \frac{c}{\lambda}$

$$\cancel{\text{and}}\ d\nu = \frac{c}{\lambda^2} d\lambda$$

Substituting $d\nu$ in Eqn (9) we get

$$E(\lambda)d\lambda = \frac{8\pi h}{c^3} \cdot \frac{c^3}{\lambda^3} \frac{c\, d\lambda}{\lambda^2} \left(\frac{1}{e^{hc/\lambda KT} - 1}\right)$$

or Energy density, $\boxed{E(\lambda)d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{(e^{hc/\lambda KT} - 1)}}$ (10)

The above Equation represents Planck's law for black body radiation in terms of wavelength (λ)

For low frequency radiation, $h\nu << KT$, then $\frac{h\nu}{KT} << 1$ and expanding the term $e^{\frac{h\nu}{KT}}$ as power series and

Neglecting the higher order terms

$$e^{\frac{h\nu}{KT}} = 1 + \frac{h\nu}{KT} + \left(\frac{h\nu}{KT}\right)^2 + \cdots \simeq 1 + \frac{h\nu}{KT}$$

Substituting in planck's law gives Eqn (9)

$$E(\nu)d\nu = \frac{8\pi h}{c^3} \frac{\nu^3}{\left(\frac{h\nu}{KT}\right)} d\nu = \frac{8\pi KT}{c^3} \nu^2 d\nu$$

$$\boxed{E(\nu)d\nu = \frac{8\pi KT}{c^3} \nu^2 d\nu} \quad \text{—— (11)}$$

This is Rayleigh - Jeans law. Therefore at low frequency approximation Planck's law reduces to Rayleigh - Jeans law. If frequency ν is expressed in terms of wavelength λ. $\nu = \frac{c}{\lambda} \Rightarrow d\nu = \frac{c}{\lambda^2} d\lambda$ substitute in Eq (10)

$$\boxed{E(\lambda)d\lambda = \frac{8\pi KT}{\lambda^4} d\lambda} \quad \text{—— (12)}$$

This is Rayleigh - Jeans law in terms of wavelength. At longer wavelength approximation Planck's law reduces to Rayleigh - Jeans law.

For high frequency radiation $h\nu >> KT$. Then $\left(e^{\frac{h\nu}{KT}} - 1\right) \simeq e^{\frac{h\nu}{KT}}$ Then Planck's law reduces to

$$\boxed{E(\nu)d\nu = \frac{8\pi h \nu^3}{c^3} e^{-\frac{h\nu}{KT}} d\nu} \quad \text{—— (13)}$$

This is Wein's law. Therefore, at high frequency approximation, Planck's law reduces to Wein's law.

If frequency γ is expressed in terms of wavelength (λ). $\gamma = \frac{c}{\lambda} \Rightarrow d\gamma = \frac{c}{\lambda^2} d\lambda$ substituting Eq (12)

$$\boxed{E(\lambda)d\lambda = \frac{8\pi hc}{\lambda^5} e^{-\frac{hc}{\lambda KT}} d\lambda} \quad (14)$$

This is Wein's law interms of wavelength (λ).

Wein's law is the shorter wavelength approximation of planck's law. This is also called as Wein's fifth power law.

Pb

1) Fermi energy of conduction electrons in silver is 5.48 eV. Calculate the number of such electrons per cm^3. Given that $h = 6.62 \times 10^{-34}$ Js and $1eV = 1.6 \times 10^{-19} J$.

Soln. Fermi energy of silver, $E_F = 5.48 eV$.

To find Number of electrons present in $1 cm^3$

$$E_F = \left(\frac{h^2}{2m}\right)\left(\frac{3N}{8\pi V}\right)^{2/3}$$

$$E_F^{3/2} = \left(\frac{h^2}{2m}\right)^{3/2}\left(\frac{3N}{8\pi V}\right)$$

$$\frac{N}{V} = \frac{(2m)^{3/2} E_F^{3/2} 8\pi}{3h^3}$$

$$= \frac{2^{3/2} \times (9.1\times 10^{-31})^{3/2} \times 8\pi (5.48 \times 1.62 \times 10^{-19})^{3/2}}{3 \times (6.62 \times 10^{-34})^3}$$

$$= 5.9 \times 10^{16} m^{-3}$$

Number of electrons / cm^3, $\frac{N}{V}$

$$= 5.9 \times 10^{22} cm^{-3}.$$

(2) The number of free electrons per c.c is 24.2×10^{22} in beryllium and 0.91×10^{22} in Cerium. If the Fermi energy of Conduction electrons in Be is 14.44 eV, Calculate that in Cerium.

Soln. Number of free electrons in Be, $n_1 = 24.20 \times 10^{22}$

in Cerium. $n_2 = 0.91 \times 10^{22}$

$E_F(Be) = 14.44\,eV$.

To find Fermi energy of free electrons in Cerium

Electrons per c.c in Be $= \frac{N}{V} = 24.2 \times 10^{22}$

Electrons per c.c in Ce $= \frac{N}{V} = 0.91 \times 10^{22}$

Fermi energy $E_F = \frac{h^2}{2m}\left(\frac{3}{8\pi}\right)^{2/3}\left(\frac{N}{V}\right)^{2/3} = K\left(\frac{N}{V}\right)^{2/3}$

$$\therefore \frac{\text{Fermi energy for Be}}{\text{Fermi energy for Ce}} = \frac{K\left(24.2 \times 10^{22}\right)^{2/3}}{K\left(0.91 \times 10^{22}\right)^{2/3}}$$

$$= \left(\frac{24.2}{0.91}\right)^{2/3}$$

$$\text{Fermi energy for } C_e = 14.44\left(\frac{0.91}{24.2}\right)^{2/3} = 1.587\,eV.$$

3) The fermi energy for lithium is 4.72 eV at T=0K. Calculate the number of conduction electrons per unit volume in lithium.

Soln. Fermi energy for lithium. $E_F = 4.72$ eV. $T = 0K$

To find electron density in lithium.

Fermi energy. $E_F = \frac{h^2}{2m}\left(\frac{3n}{8\pi}\right)^{2/3}$ per unit volume $V = 1m^3$.

Electron density, $n = \frac{8\pi}{3}\left[\frac{2mE_F}{h^2}\right]^{3/2}$

$$= \frac{8\times 3.14}{3}\left[\frac{2\times 9.11\times 10^{-31}\times 4.72\times 1.6\times 10^{-19}}{(6.62\times 10^{-34})^2}\right]^{3/2}$$

$n = 2.06\times 10^{27}$ per m^3.

4) The fermi energy for silver is 5.50 eV. Calculate the fermi temperature and fermi velocity.

$$T_F = \frac{E_F}{K} = \frac{5.50\times 1.6\times 10^{-19}}{1.38\times 10^{-23}}$$

$K = 1.38\times 10^{-23}$ J/K

$= 6.377\times 10^{4}$ K

Fermi velocity $V_F = \sqrt{\frac{2\times 5.5\times 1.6\times 10^{-19}}{9.1\times 10^{-31}}}$

$= 1.39\times 10^{6}\ ms^{-1}$.

5) Calculate the probability that in tossing a coin 6 times, we get 3 heads and 3 tails.

Soln tossing a coin 6 times is equivalent to 6 similar coins simultaneously. Hence $n = 6$.

$r = 3$ $n - r = 6 - 3 = 3$

The probability of distribution $(r, n-r)$ is given by

$$P(r, n-r) = \frac{1}{2^n}\,{}^nC_r = \frac{1}{2^n}\frac{n!}{r!(n-r)!}$$

$= \frac{1}{2^n} \frac{n!}{r!\, n-r!}$ $2\times2\times2\times2\times2\times2$

$= \frac{6!}{2^6\, 3!\cdot 3!}$ $= \frac{6!}{2^6 \times 3! \times 3!}$

$= \frac{2\,\cancel{6}\times5\times\cancel{4}^{2}\times\cancel{3\times2\times1}}{64\ \cancel{729}\times\cancel{3}\times\cancel{7}\times1\times\cancel{3\times2\times1}}$

$= \frac{\cancel{20}}{\cancel{729}}$

(6) Calculate the probability ~~that~~ in tossing a coin 10 times, we get i) all heads (ii) 5 heads and 5 tails and (iii) 3 heads & 7 tails iv) 7 heads & 3 tails

Soln: The probability of distribution $(r.\ n-r)$ is given by

$P(r.\ n-r) = \frac{1}{2^n}\, nC_r = \frac{1}{2^n} \frac{n!}{r!\, n-r!}$

tossing a coin 10 times is equivalent to 10 similar coins.

i) tossing a coin 10 times

Here $n = 10$ and $n-r = 0$ $\quad 0! = 1.$

$\therefore r = 10$

$\therefore P(10,0) = \frac{1}{2^{10}}\, nC_r = \frac{1}{2^{10}} \frac{10!}{10!\, 0!} = 1$

$\therefore P(10,0) = \frac{1}{2^{10}} = \frac{1}{1024} = 0.00098$

(ii) We wish to have 5 heads and 5 tails.

$\therefore r = 5.$ and $n - r = 5$ tails

$P(5.5) = \frac{1}{2^{10}}\, 10C_5 = \frac{1}{2^{10}} \frac{10!}{5!\ 5!} = \frac{10!}{2^{10}\, 5! \times 5!}$

$= \frac{252}{1024} = 0.24609$

(iii) We have 3 heads & 7 tails

$r = 3$ & $n - r = 10 - 3 = 7.$

$P(3.7) = \frac{1}{2^{10}}\, 10C_3 = \frac{1}{1024} \times \frac{10!}{3!\,7!} = \frac{120}{1024}$

$= 0.11719$

iv) we have 7 heads & 3 tails.

$r = 7$ and $n - r = 3$

$$P(7.3) = \frac{1}{2^{10}} {}^{10}C_3 = \frac{1}{2^{10}} \frac{10!}{7!\,3!}$$

$$= 0.11719$$

Thus the probability of distribution (7, 3) is equal to probability of distribution (3.7)