Self induction: It is the Property of the coil which opposes any changes in current Passing through it.

co-efficient of self induction:- (self induction)

$$\phi \propto I$$

$\phi = LI$ where $L \rightarrow$ coeff. of self induction.

If $I = 1A$ then $\boxed{\phi = L}$

The self inductance of a coil is numerically equal to the mag.flux linked with it when a current of 1A flows through it.

$$\frac{d\phi}{dt} = L\frac{dI}{dt}$$

$\therefore\ e = -L\frac{dI}{dt}$ If $\frac{dI}{dt} = 1As^{-1}$ then

$$\boxed{e = L}$$

$\therefore$ Self inductance of a coil is numerically equal to emf induced in the coil when current in it changes at the rate of $1As^{-1}$

S.I. unit of Self inductance is henry (H)

Define henry:- $e = L\frac{dI}{dt}$

If $\frac{dI}{dt} = 1As^{-1}$, $e = 1$ volt then

$$\boxed{L = 1H}$$

The self inductance of a coil is said to be 1 henry when a current in it changing at the rate of $1As^{-1}$, induces an emf of 1V in the coil.

Expression for self inductance of a coil :-

Consider a coil of length l, area A and turns N. let I be the current flowing Then mag. field is given by $B = \frac{\mu_0 N I}{l}$ tesla —— (1)

Total flux linked with the coil $\phi = NBA$

$$\phi = N\left(\frac{\mu_0 N I}{l}\right)A = \frac{\mu_0 N^2 I A}{l} \quad \text{—— (2)}$$

By definition $\phi = L I$ —— (3)

where L → self inductance of the coil

Equating eqns (2) & (3)

$$L = \frac{\mu_0 N^2 A}{l} \text{ henry}$$

where $\mu_0 = 4\pi \times 10^{-7}\,Hm^{-1}$

If the coil is wound on material of relative permeability μ_r then

$$\boxed{L = \frac{\mu_0 \mu_r N^2 A}{l}}$$

Energy stored in a coil :

Consider a coil of inductance 'L' in a circuit, let the current grows from zero to maximum value I_0

The induced emf $E = -L\frac{dI}{dt}$

This induced emf tends to prevent the growth of current. To maintain the growth,

work has to be done against the back emf.
The work done in small interval of time dt is

$$dW = -EI\,dt$$

$$dW = -\left[-L\frac{dI}{dt}\right] I\,dt$$

$$dW = LI\,dI$$

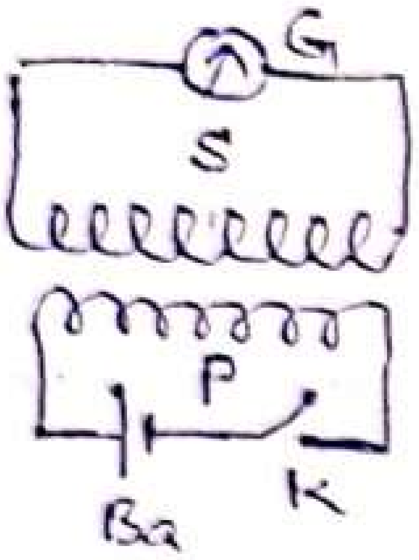
Total work done $W = \int_0^{I_0} LI\,dI$

$$W = \left[\frac{LI^2}{2}\right]_0^{I_0}$$

$$W = \frac{1}{2}LI_0^2$$

This work done is stored as energy in the form of magnetic field $\therefore \boxed{E = \frac{1}{2}LI_0^2}$

Mutual induction :-



Consider a secondary coil S placed in the mag. field of the primary coil P. when the current in the coil P increases, mag flux linked with coil S also increases and vice versa. The galvanometer shows deflection. This shows that an emf and current is induced in coil S due to change of current in the coil P.

$\therefore$ Mutual induction is the phenomenon of inducing emf in one coil due to current changes in a neighbouring coil.

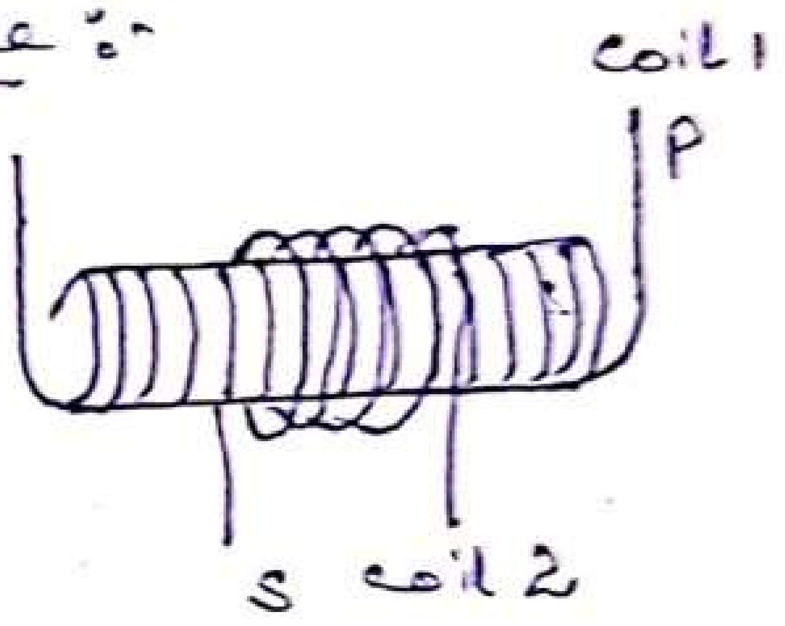
Mutual inductance or coefficient of mutual induction:-

Consider $e = M \frac{dI}{dt}$

when $\frac{dI}{dt} = 1\,A s^{-1}$ then $\boxed{M = e}$

Mutual inductance between pair of coils is numerically equal to the emf induced in one coil when current changes at the rate of one ampere per second in the neighbouring coil. S.I. unit of mutual inductance is henry (H)

Expression for mutual inductance :-



Consider a primary coil P of length L, area A and no of turns N_p. Consider a short secondary coil S of turns N_S wound closely over central portion of coil P.

If I be the current flowing in P then mag. field

$$B = \frac{\mu_0 N_p I}{l}$$

flux linked in primary P is $\phi' = BA$

$$\phi' = \frac{\mu_0 N_p I A}{l} \quad \text{——(1)}$$

Total flux linked in secondary S is $\phi = N_S \phi'$ ——(2)

$$\phi = \frac{\mu_0 N_p N_S I A}{l} \quad \text{——(3)}$$

But $\phi = MI$ ——(4) comparing eqn (3) & (4)

$$\boxed{M = \frac{\mu_0 N_p N_S A}{l}}$$

If the coil is wound material of relative permeability then

$$\boxed{M = \frac{\mu_0 \mu_r N_p N_S A}{l}}$$

Eddy current:- when a metal block like copper disc is in relative motion with a magnetic field, induced current are set up in it, opposing the relative motion. These induced currents are called eddy currents

Applications:

1) **Electromagnetic damping:** In a moving coil galvanometer the coil is wound on a metal frame. when the frame rotates with the coil in the mag. field, eddy currents are formed in it opposing its motion and thus the coil is quickly brought to rest.

2) **Induction furnace:** A high frequency alternating current is passed through a circuit which sarrounds the metal furnace the eddy currents are produced in the metal produce high temp.

3) **Electric breakes:** The axle of train is fitted with metal drums. These drums rotates along with the axle. when train is to be stopped, a strong magnetic field is applied on these drums using electromagnet. The eddy current generated in the drums opposes the motion of axle. Hence the train is brought to rest.

4) Speedometer :- A magnet is placed inside an aluminium drum and is carefully pivoted and held in position by hair spring. This magnet revolves according to the speed of the vehicle. Eddy currents in the drum are set up and drag it through an angle which depends on the speed of the vehicle.

5) Induction motor: A metallic cylinder is placed in a rotating magnetic field and the eddy current produced in the metallic cylinder decreases the relative motion between the cylinder and the rotating mag. field. As the mag. field is continuosly rotated metallic cylinder is set up in rotation due eddy current.

Disadvantages of Eddy currents:

1) Eddy currents dissipate electrical energy in the form of heat in transformer, dynamo etc.

2) Heat produced due to eddy currents breaks the insulation in the electric circuits.

1st B.Sc. 2nd sem   Unit - III   S.G. Rangaswamy.
PHYSICS

⊙ Derivation for paramagnetic susceptibility (or) Curie's law (or) Langevin's theory of paramagnetism :

Paramagnetic substances are characterized by a molecules having a permanent magnetic moments. As the molecules have permanent dipole moment, they do get aligned in an external field.

Consider a paramagnetic substance having 'N' molecules (or) atoms per unit volume at a temperature 'T'. Each atom (or) molecule is assumed to possess a permanent magnetic moment $\vec{m}$. If all the dipoles are along the applied magnetic field, then the total magnetization ($\vec{I}$) of all dipoles will be $N\vec{m}$. The magnetic potential energy of the dipole which is inclined at an angle 'θ' with the applied field $\vec{B}$ is given by

$$U = -\vec{m}\cdot\vec{B} = -mB\cos\theta \longrightarrow ①$$

From Boltzman distribution law, the relative probability of finding a dipole making an angle 'θ' with the applied $\vec{B}$ is given by

$e^{-U/KT}$ where K = Boltzman Constant.

Let 'dN' be the no of dipoles inclined b/w the angles θ and θ+dθ with the field $\vec{B}$.

Then $dN \propto e^{-U/KT}\, d\Omega$

$$dN = Ae^{-U/KT}\, d\Omega \longrightarrow ②$$

where 'A' is a constant of proportionality and $d\Omega$ is the solid angle subtended by the cones of semi-vertical angle θ and θ+dθ and the solid angle is given by $d\Omega = 2\pi\sin\theta\, d\theta$

$$\therefore dN = Ae^{-U/KT}\cdot 2\pi\sin\theta\, d\theta$$

Using Equation ① $dN = Ae^{(mB\cos\theta)/KT}\cdot 2\pi\sin\theta\, d\theta$

$$dN = 2\pi Ae^{\alpha\cos\theta}\sin\theta\, d\theta \longrightarrow ③$$

here $\alpha = mB/KT$

Total no of dipoles per unit volume (N) can be obtained by

Integrating the Equation ③ b/w the limits '0' and 'π'

ie $N = \int_0^\pi dN = \int_0^\pi 2\pi A e^{\alpha\cos\theta} \sin\theta \, d\theta$

$$N = 2\pi A \int_0^\pi e^{\alpha\cos\theta} \sin\theta \, d\theta$$

But $\int_0^\pi e^{\alpha\cos\theta} \sin\theta \, d\theta = \frac{2\sinh\alpha}{\alpha}$

$$\therefore N = \frac{4\pi A \sinh\alpha}{\alpha}$$

(or)

$$A = N\alpha / 4\pi \sinh\alpha \longrightarrow ④$$

Using the Value of 'A' in Equation ③ we get

$$dN = \frac{N\alpha}{2\pi \sinh\alpha} \left(e^{\alpha\cos\theta}\right) \sin\theta \, d\theta \longrightarrow ⑤$$

The Component of Each dipole moment parallel to applied field $\vec{B}$ is $m\cos\theta$, hence the total magnetic moment per unit volume (or) total magnetization of the substance is given by

$$I = \int_0^\pi m\cos\theta \, dN$$

Using Equation ⑤

$$I = \frac{mN\alpha}{2\sinh\alpha} \int_0^\pi e^{\alpha\cos\theta} \sin\theta \cos\theta \, d\theta$$

$$I = \frac{mN\alpha}{2\sinh\alpha} \left[\frac{e^{\alpha\cos\theta}}{\alpha^2} - \frac{\cos\theta \, e^{\alpha\cos\theta}}{\alpha} \right]_0^\pi$$

$$I = \frac{mN}{2\sinh\alpha} \left[\frac{e^{\alpha\cos\theta}}{\alpha} - \cos\theta \, e^{\alpha\cos\theta} \right]_0^\pi$$

$$I = \frac{mN}{2\sinh\alpha} \left[\frac{e^{-\alpha}}{\alpha} + e^{-\alpha} - \frac{e^{\alpha}}{\alpha} + e^{\alpha} \right]$$

on simplifying above Equation, we get

$$I = Nm \left(\coth\alpha - \frac{1}{\alpha}\right) \longrightarrow ⑥$$

Since Nm is the saturation value of the magnetic moment

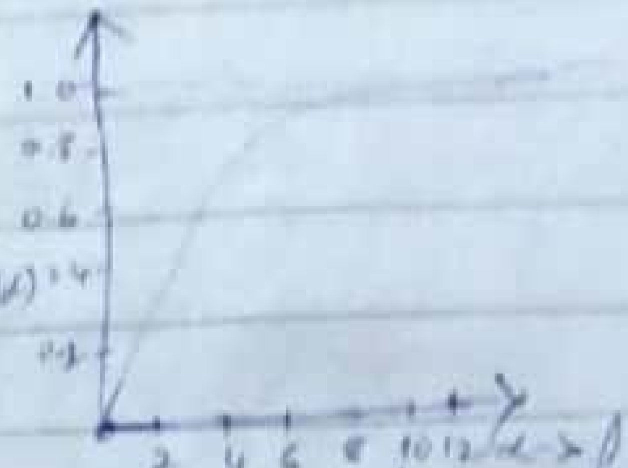
hence $Nm = I_0$.

ie, when the substance is completely aligned with the applied field then the magnetic moment is known as saturation magnetic moment (I_0)

$$\therefore I = I_0 \left(\cot\alpha - \frac{1}{\alpha}\right)$$

(or) $I = I_0 L(\alpha)$ → ⑦

where $L(\alpha) = [\coth\alpha - \frac{1}{\alpha}]$ is known as Langevin function, variation of Langevin function $L(\alpha)$ with α is as shown in fig. It is observed from the plot that, for large value of α the Langevin function tends to Unity $I = I_0$. In this case all atomic dipoles are parallel to $\vec{B}$ and magnetisation attains saturation. For a small value α the curve is linear and for smaller value of α the Langevin function becomes



Variation of Langevin func.

$$L(\alpha) = \coth\alpha - \frac{1}{\alpha} = \alpha/3$$

Using α values $L(\alpha) = \frac{mB}{3kT}$

Using this value in Eqn ⑦ $I = I_0 m B/3KT$ but $I_0 = Nm$

$$\therefore I = \frac{Nm^2 B}{3KT} \rightarrow ⑧$$

For paramagnetic substance, I is very small positive value, hence $B = \mu_0(H + I) = \mu_0 H$

Using this value in Equation ⑧

$$\therefore I = \frac{Nm^2 \mu_0 H}{3kT} \rightarrow ⑨$$

By the definition of magnetic susceptibility

$$\chi = \frac{I}{H} = \frac{Nm^2 \mu_0 H}{3kTH} = \frac{\mu_0 N^2 m^2}{3NKT}$$

$$\chi = \frac{\mu_0 M_0^2}{3NkT} = \frac{\text{constant}}{T} = \frac{C}{T}$$

$$\chi = C/T \rightarrow ⑩$$

hence $C = \mu_0 M_0^2/3Nk$ is a constant called Curie Constant. Equation ⑩ is known as Curie law. It states that, the magnetic susceptibility of a paramagnetic substance varies inversely with the absolute temperature.

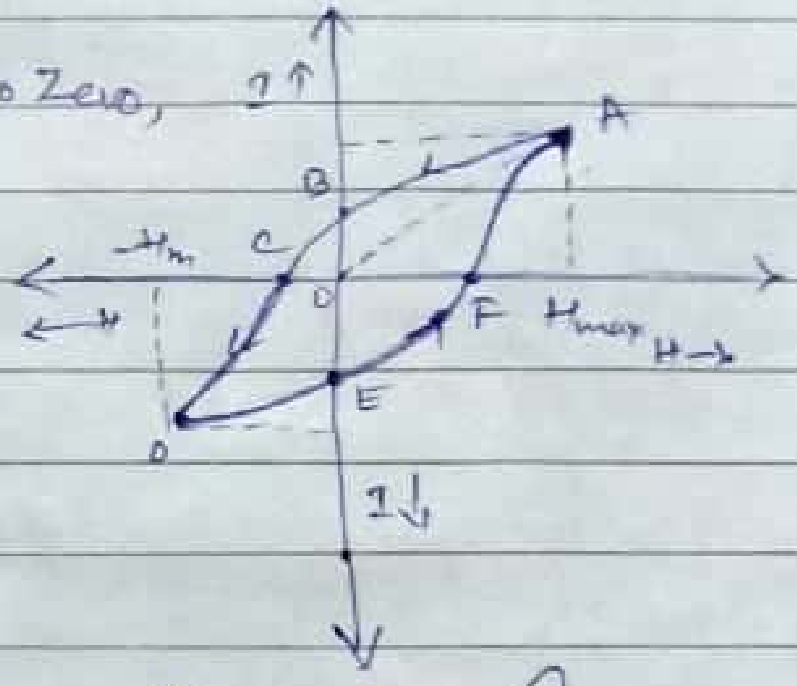
⊛ Hysteresis:

The term hysteresis is derived from a Greek word meaning "Lagging behind". The lagging of intensity of magnetisation behind the magnetic field is known as hysteresis.

⊛ Hysteresis Curve:

Consider an Unmagnetized ferromagnetic material say iron bar and it is subjected to a gradually increasing magnetising field H. When H=0, I=0 and at 'O' in fig. As 'H' increases gradually, 'I' also increases in a non-linear way and attains a saturation at 'A' is as shown fig. This is represented by a Curve 'OA' called the initial magnetization curve. At this stage, all the dipoles are aligned and 'I' has reached to its maximum (or) saturated value.

As 'H' decreases from Hmax to Zero, it is found that the value of 'I' also decreases but at a lower rate and follows the path AB. When 'H' is reduced to Zero, 'I' is not come to its zero value. but it has certain value in it and is represented by OB.



Hysteresis Cure

The magnetization left behind in the specimen even when the magnetizing field is made zero is called residual magnetism (or) remanence. It gives the state of permanent magnetization of the specimen.

On applying the reverse field, the value of 'I' falls and finally becomes zero. The abscissa 'OC' represents the reverse magnetizing field needed to reduce magnetization to zero (or) to completely demagnetize the specimen and the field is known as Coercive field. If the reverse magnetizing field is further increased, the magnetization increases in the reverse direction and attains saturation at D. Again if the field is reduced to zero, 'I' follows the path DE & becomes equal to OE when H = 0. here OE represents the residual magnetism.

When the field is again increased from zero to H_{max}, the curve EFA is traced and the saturation is again attained in the +ve direction at A. The closed loop ABCDEFA represents the hysteresis curve of the material. It is observed that throughout the cycle of magnetization, I lags behind H. this lagging behind of 'I' with H is known as hysteresis.

⑧ Significance of hysteresis curve :-

- → It provides information regarding to the magnetic properties of the substance and helps to select suitable magnetic materials for various practical purposes.
- → Hysteresis loops are different for different materials and one can compare the properties of the materials from the ~~hys~~ hysteresis loop.
- → The area of the hysteresis loop gives the amount of energy lost during one cycle of magnetization.
- → The choice of magnetic materials for different uses can be decided from the hysteresis curve of a specimen of the material.

⑨ Problems :-

① A bar magnet made of steel has a magnetic moment of 2.5 Am^2 and a mass of 6.6×10^{-3} kg. If the density of steel is 7.9×10^{3} kgm^{-3}, find the intensity of magnetization of the magnet.

Given data : magnetic moment (M) = 2.5 Am^2

mass (m) = 6.6×10^{-3} kg

density (ρ) = 7.9×10^{3} $kg\,m^{-3}$

Horizon

Sol:- We have, the Intensity of magnetisation $I = \frac{m}{V}$
where V is the volume of material

and $V = \frac{\text{mass}}{\text{density}}$

$V = \frac{6.6\times10^{-3}}{7.9\times10^{3}}$

$V = 8.3\times10^{-7}\,m^3$

$I = \frac{2.5}{8.3\times10^{-7}}$

$\boxed{I = 3\times10^{6}\,Am^{-1}}$

⑧ Concept Questions:-

① Is it possible to obtain an isolated north ⓐ South pole by breaking a magnet into two pieces? why?

Ans: No.

Because according to Gauss law of magnetism isolated magnetic pole does not Exist and magnetic flux through any closed surface is zero.

② Which are the two quantities related to magnetism have same dimensions?

Ans: Intensity of magnetisation and magnetizing field intensity.

③ Can we have a magnetic hysteresis in paramagnetic ⓞ diamagnetic substance? why?

Ans: No

because the hysteresis is a special property Exhibited by only ferromagnetic materials.