

Sree Siddaganga College of Arts, Science and Commerce , B.H Road,Tumkur
DEPARTMENT OF MATHEMATICS

Mathematics Question Bank

III BSc VI Semester Paper -6.1 Complex and Numerical analysis.

TWO Marks Questions

- 1) Find locus of a point z , satisfying $|z + i| \leq 2$.
- 2) Show that $|z - (2 + 3i)| = 5$ represents a circle.
- 3) Evaluate $\lim_{z \rightarrow i} \left(\frac{z^2+1}{z^6+1} \right)$.
- 4) Show that $f(z) = z^2+2z$ is continuous at $(i+i)$.
- 5) Show that $u = x^3 - 3xy^2$ is harmonic.
- 6) Evaluate $\lim_{z \rightarrow 2e^{i\pi/3}} \frac{z^3+8}{z^4+4z^2+16}$
- 7) Find real and imaginary parts of the function $\log_e z$.
- 8) Show that $\cosh z$ is analytic.
- 9) Show that for the function $f(z) = \bar{z}$, $f'(z)$ do not exists.
- 10) Prove that $E=e^{hD}$ where ' h ' is the interval of difference.
- 11) If $v_0=3, v_1=12, v_2=81, v_3=200, v_4=100, v_5=8$ find value of $\Delta^5 v_0$
- 12) Find the n^{th} difference of $\sin(ax+b)$.
- 13) Find the real and imaginary parts of $e^{\frac{i\pi}{2}}$.
- 14) Show that $u = e^x \sin y$ is a Harmonic function.
- 15) State cauchy's inequality .
- 16) Define Power Series.
- 17) Evaluate $\int_{(0,1)}^{(2,5)} ((3x + y)dx + (2y - x)dy)$ along the curve $y = x^2 + 1$.
- 18) Evaluate $\int_c \frac{e^z}{(z-1)} dz$ where $c: |z| = 2$.
- 19) Prove that $\Delta = E - 1$.
- 20) Evaluate $\Delta^3 [(1 + 2x)(1 + 4x)(1 + 6x)]$ by taking $h=1$.

THREE Marks Questions

- 1) Show that $\arg. \left| \frac{z-1}{z+2} \right| = \frac{\pi}{3}$ represents a circle.
- 2) Show that $u = x^3 - 3xy^2$ is harmonic .
- 3) Find the orthogonal trajectories of family of curves $x^2-y^2 = C$.
- 4) Evaluate $\oint \frac{z}{(z^2+1)(z^2-9)} dz$, where c is the circle $|z| = 2$
- 5) Verify C R equations for the function $W=\sin z$,
- 6) Find the n th difference of a^{bx+c} .
- 7) Show that every differentiable function is continuous.
- 8) If $u(x,y)$ and $v(x,y)$ are harmonic Conjugate of each other then Prove that they are Constant .

- 9) Show that the function $f(z) = \frac{x(x-iy)}{x^2+y^2}$ is not analytic at the origin.
- 10) Find analytic function where imaginary parts of $e^x \sin y$.
- 11) Evaluate $\lim_{z \rightarrow e^{\frac{i\pi}{4}}} \frac{z^2}{z^4+z^2+1}$.
- 12) State Liouville's theorem
- 13) Verify whether $(z) = z - \bar{z}$, is differentiable or not using C R equations.
- 14) Show that $\int_c \frac{dz}{z-a} = 2\pi i$, where c is a circle $|z - a| = r$.
- 15) Evaluate $\lim_{z \rightarrow i} \frac{iz^3-1}{z+i}$.

FIVE Marks Questions

UNIT-1

- 1) Show that every differentiable function is continuous.
- 2) If $u(x,y)$ and $v(x,y)$ are harmonic Conjugate of each other then Prove that they are Constant
- 3) Show that the function $f(z) = \frac{x(x-iy)}{x^2+y^2}$ is not analytic at the origin.
- 4) Find analytic function where imaginary parts of $e^x \sin y$.
- 5) Derive C-R equation for the function $f(z)$.
- 6) Show that $2u = \log(x^2+y^2)$ for $x \neq 0$, $y \neq 0$ is harmonic and find its harmonic conjugate.
- 7) If $f(z) = u(r, \theta) + i v(r, \theta)$ is an analytic function, then show that

$$\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} + \frac{1}{r^2} \frac{\partial^2 v}{\partial \theta^2} = 0.$$

- 8) Find the analytic function $f(z) = u + i v$, given $u = \frac{\cos \theta}{r}$.
- 9) State and prove the necessary condition for $f(z) = u + iv$ to be an analytic function
- 10) Show that $\arg. \left| \frac{z-1}{z+2} \right| = \frac{\pi}{3}$ represents a circle. Find its centre and radius.
- 11) Show that $u = x^3 - 3xy^2$ is harmonic and find its harmonic conjugate.
- 12) Find the orthogonal trajectories of family of curves $x^2 - y^2 = C$.
- 13) State and Prove Cauchy's theorem for a simply connected region.
- 14) Construct the analytic function whose imaginary part is $v = e^x(x \sin y + y \cos y)$.
- 15) State and Prove sufficient condition for the function $f(z) = u + iv$ to be analytic in a domain D of the complex plane.

UNIT-2

- 1) State and prove Cauchy's - integral formula.
- 2) State and prove Cauchy's inequality .
- 3) State and prove Liouville's theorem .
- 4) State and prove Cauchy's theorem for a simply connected region .
- 5) State and Prove Fundamental theorem of algebra.
- 6) Evaluate $\int_c (x^2 - iy^2) dz$ along the curve $y = 2x^2$ from (1,1) to (2,8).

- 7) Evaluate $\int_0^{1+i} (x^2 - iz) dz$ along the line $y = x$ and $y = x^2$.
- 8) Evaluate $\int_c \frac{\sin(\pi z^2) + \cos(\pi z^2)}{(z-1)(z-2)} dz$ where $c: |z| = 3$
- 9) Evaluate $\int_c \frac{(z+4)}{(z^2+2z+8)} dz$ where c is a circle $|z + 1 + i| = 2$
- 10) Evaluate $\int_c \frac{e^{ax}}{z^2+1} dz$ where c is the circle $|z| = 2$.
- 11) Evaluate $\int_c \frac{\sin^2 z}{(z-\frac{\pi}{6})^3} dz$ where $c: |z| = 1$.
- 12) If $f(a) = \oint_c \frac{z^2+3z+2}{z-a} dz$ where c is the ellipse $9x^2 + 16y^2 = 144$, Find $f(1), f(-2)$.
- 13) Evaluate $\oint_c \frac{z}{(z^2+1)(z-2)} dz$, where c is the circle $|z| = 2$.
- 14) Evaluate $\oint_c \frac{z}{(z^2+1)(z^2-9)} dz$, where c is the circle $|z| = 2$.
- 15) Evaluate $\int_c \frac{\sin(\pi z^2) + \cos(\pi z^2)}{(z-1)^2(z-2)} dz$ where $c: |z| = 1.5$.
- 16) Evaluate $\int_c \frac{(z-1)}{(z+1)^2(z-2)} dz$ where c is a circle $|z - i| = 2$.

UNIT-3

- 1) Find a Polynomial of degree 3 which passes through the (0,3), (1,6), (2,11), (3,24) and (4,51).
- 2) If $f(x)$ be a polynomial of n^{th} degree in x , prove that $\Delta^n f(x)$ is a constant and $\Delta^{n+1} [f(x)] = 0$.
- 3) From the difference table, find the 8th term of sequence 7, 15, 35, 72, 131, 217,
- 4) By constructing difference table, find the 10th term of sequence 3, 14, 39, 84, 155, 258,
- 5) From the table, find the value of $e^{0.24}$

X	0.1	0.2	0.3	0.4	0.5
Y	1.10517	1.22140	1.34986	1.49182	1.64872

- 6) Given,

X	5	7	11	13	17
Y	150	392	1452	2366	5202

Evaluate $Y(9)$ Using Lagrange's method

- 7) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $X = 51$ from the following data

X	50	60	70	80	90
Y	19.96	36.65	58.81	77.21	94.61

- 8) The population of a town is as follows, Find increase in population during the period 1953 to 1961.

Year	1921	1931	1941	1951	1961	1971
Population (lakhs)	20	24	29	36	46	51

9) Find $\log_{10} 303$ for the following data

X	300	304	305	307
$\log_{10} X$	2.4771	2.4829	2.4843	2.4871

10) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the given that $X=2.2$

X	1.4	1.6	1.8	2.0	2.2
Y	4.0552	4.9530	6.0496	7.3891	9.0250

11) Estimate the Population in 1996 from the table.

Year	1992	1993	1994	1995	1996	1997
Population in lakhs	72	106	146	192	-	302

12) Evaluate $\int_0^1 \frac{x}{(1+x^2)} dx$ using simpson's (3/8)th rule by dividing interval in to 3 equal parts and hence find an approximate value of $\log \sqrt{2}$.

13) Evaluate $\int_0^1 \frac{dx}{(1+x^2)}$ using simpson's (1/3)th rule by dividing the range in to 6 equal parts.

Hence find an approximate value of π .

14) Evaluate $\int_4^{5.2} \log_e x dx$ using wedlle's

15) Drive general quadrature formula and hence obtain simpson's (1/3)th rule.
