Sree Siddaganga College of Arts, Science and Commerce , B.H Road,Tumkur

DEPARTMENT OF MATHEMATICS

Mathematics Question Bank

III BSc VI Semester Paper -6.2a Number Theory.

TWO Marks Questions

1) Define divisibility of two integers. Give an example.
2) If a/b and 'x' is any integer then prove that a/bx and Give an example.
3) State gcd of two integers and find the gcd of -245 and 17.
4) Find the remainder when 2^{100} is divided by 5.
5) Define congruence relation. Give an example.
6) State Chinese remainder theorem.
7) Prove that the square of an odd integer is of the from 8q+1.
8) If $3(3x+1) \equiv x+3 (mod 4)$ Find 'x'.
9) If a/b and a/c for any integer m and n then Show that $a/(bm+cn)$.
10) Solve 5x≡4(mod 13)..
11) If an integer '**b**' divides '**a**' positive integer '**a**' then prove that '**b**' is not numerically greater than '**a**'.
12) If $d = (a, b)$ then show that $\left(\frac{a}{d}, \frac{b}{d}\right) = 1$.
13) Find the unit digit in the number 7^{126}.
14) If $a \equiv b (mod m) and\ x$ is an any integer then P.T $(a+x) = (b+x)(mod m)$.
15) Define multiplicative function. Give an example of multiplicative function.
16) Define Sum and Number of divisors function
17) Define a mobius function. Give an example.
18) Find the product of all the positive divisors of 68.
19) If f is a multiplicative function then Prove that $f(1) = 1$
20) Find a Positive integer a such that $\mu(n) + \mu(n+1) + \mu(n+2) = 3$.

THREE Marks Questions

1) Find the number and sum of all positive divisors of 960.
2) If there exists 'x' and ' y' integers such that ax+by=1 then show that (x, y) =1
3) Prove that the relation a≡b(mod m) is an equivalence relation.
4) If $(a, b) = d$ then the equation $ax + by = c$ has a solution then prove that d/c
5) Let 'n' be an integer > 1, then Show that $\tau(n)$ is odd iff 'n' is a perfect square.
6) Show that 'τ' function and 'σ' function are multiplicative function.
7) If a/bc and $(a, b) = 1$ then P.T a/c.

8) If a/b and a/c then show that $a/(b-c)$ and a/bc.
9) Solve $5x \equiv 4(mod 13)$.
10) If $ca \equiv cb(modm)$ and $(c,m) = 1$ then $a \equiv b(modm)$.
11) If '**p**' is a prime and '**a**' positive integer then $\emptyset(p^a) = p^p - p^{a-1}$ and find $\emptyset(2^{10})$.
12) If a,b and c are integers show that $ac \equiv bc(modm) and (c,m) = 1$ then Prove that $a \equiv b(modm)$
13) Solve 2x≡1(mod 7) and x≡1(mod 5).
14) If (m,n)=1 then Prove that φ is multiplicative.
15) If p is a prime and k>0 then P T $\varphi(p^k) = p^k\left(1 - \frac{1}{p}\right) = p^k - p^{k-1}$.

FIVE Marks Questions

UNIT-1

1) Find the gcd of 726 and 275, and express it as $275x + 726y$.
2) If 'p' is a prime number and 'a' is any integer then prove that (a,p)=p and (a,p)=1.
3) Find the gcd of 275 and 726 and express it as $275x + 726y$.
4) Prove that for any two integers **a** and **b**>0, then there exits q_1 and $r_1 \in z$
5) Such that $a = bq_1 + cr_1, 0 \le r_1 < \frac{b}{2}$ and $e = \pm 1$.
6) Find the gcd and lcm of 119 and 272. Also express the gcd as a linear combination of 119 and 272
7) Find the number of positive divisors and their sum of a number 960.
8) State and Prove Euclid's Lemma. When Euclid's Lemma fails.
9) Find the general Solution of 70x+112y=168.
10) Find the general Solution of 170x-455y=625..
11) If a,b are any two integers both are non-zero and k is any integer then PT (ka,kb)=$|k|(a,b)$.
12) If a and b are any two integers not both zero then GCD(a,b) exists and is unique
13) Prove that one of every three consecutive integers is divisible by 3.
14) State and Prove Fundamental theorem of arithmetic.

UNIT-2

1) State and prove Chinese remainder theorem.
2) Prove that the linear congruence $ax \equiv b(modm)$ has'd' distinct solutions if $(a,m) = 1$ and d/b.
3) Show that $5^{16} - 3^{16}$ is divisible by 17.
4) Find the remainder 64*65*66 is divided by 67.
5) Prove that Linear congruence $ax \equiv b(modm)$ has a unique solution iff (a,m)=1.

6) If a, band c are integers such that $ac \equiv bc(mod m)$ where m>0 is affixed integer and $d = (c, m)$ $then\ P\ T\ a \equiv b(mod \frac{m}{d})$.
7) Find the remainder when $3^{12} + 5^{12}$ is divided by 13.
8) State and Prove willson's theorem.
9) Show that 1729 is an absolute pseudo prime number
10) What is Fermat's Number? S T Fermat's Number F_s is divisible by 641.
11) Find the remainder when 2^{340} is divided by 341.
12) If $ax \equiv bc(mod m)$, (a,m)=d and d/b then P T the linear congruence has exactly 'd' incongruent solution (modm)
13) Convert $(58.3125)_{10}$ to binary number.
14) Find 'x' which simultaneously satisfy the congruence's $x \equiv 1(mod 3),\ x \equiv 2(mod 4), x \equiv 3(mod 5)$.

UNIT-3

1) If $n > 1, n = p_1^{\alpha_1}.p_2^{\alpha_2} \dots \dots \dots \dots p_k^{\alpha_k}$ is the prime power factorization of the positive integer n, then $\frac{\emptyset(n)}{n} = \left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right) \dots \dots \dots \dots \left(1 - \frac{1}{p_k}\right)$.and find $\emptyset(246)$.
2) Find the sum and number of all positive divisors of 56700.
3) Let 'n' be a positive integer. Derive the Formula for $\tau(n)$ and $\sigma(n)$. where $\tau(n)$ is the number of positive divisors of 'n' and $\sigma(n)$ is the sum of the positive divisors of 'n'.
4) Show that mobius function $\mu(n)$ is Multiplicative function.
5) Show that arithmetic functions σ and τ are Multiplicative.
6) Evaluate σ and τ for n= 3655.
7) Show that $\frac{(2n)!}{n!^2}$ is an even integer.
8) Find the highest 18 contained in 500!.
9) If $f(n) = n^2 + 2$ and $n = 6$, then ST $\sum_{d/6} f(d) = \sum_{d/6} f(\frac{6}{d})$.
10) If m and n relatively prime then $\emptyset(m, n) = \emptyset(m).\emptyset(n)$, $\emptyset$ is Multiplicative.
11) ST $\emptyset(n) = \emptyset(n+1) = \emptyset(n+2)$ for n=5186.
12) State and Prove Euler's Generalization of Fermats's theorem
13) If $'\boldsymbol{f}'$ is a multiplicative function and $F(n) = \sum_{d/n} f(d)$ then F is also multiplicative function.
14) If gcd(m,n)=1, then Show that the set of positive divisors of 'mn' consists of all products $d_1 d_2$, $where$ d_1/m , d_2/n and gcd $(d_1, d_2) = 1$.

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