Sree Siddaganga College of Arts, Science and Commerce , B.H Road,Tumkur

DEPARTMENT OF MATHEMATICS

Mathematics Question Bank

II BSc IV Semester Paper -4.1 Alegbra and Calculus -2

TWO Marks Questions

1) Evaluate $\int_0^1 \int_0^1 \frac{dxdy}{\sqrt{(1-x^2)((1-y^2)}}$

2) Evaluate $\int_{-1}^{1} (x^2 - 3x + 1)\, dx$.

3) Evaluate $\int_c (3x + y)\, dx + (2y - x)dy,$ along the curve $y = x^2 + 1$ from (0,1) to (3,10).

4) Evaluate $\int_0^2 \int_0^{\pi} r^2 sin\theta\; d\theta dr$

5) Find the unit normal vector to the surface $(x-1)^2 + y^2 + (z+2)^2 = 9$ at (3,1,-4).

6) If the Vector $\vec{F}$ =(3x+3y+4z)i + (ax-2y+3z) j + (3x+2y-z)k is a solenoidal , Find 'a' .

7) Prove that $div(grad\varphi) = \nabla^2\varphi$.

8) Find the directional derivative of $\varphi = xy^2 + yz^3$ at (2,-1,1) in the direction of $2\hat{\imath}+\hat{\jmath}+2\hat{k}$.

9) Prove that $\nabla(\varphi\theta) = \varphi(\nabla\theta) + \theta(\nabla\varphi)$ where φ *and* θ are functions of x,y and z .

10) Define a group. give an example.

11) Define order of an element of a group.

12) Find the order of each element of the multiplicative group G=$\{1, -1, i, -i\}$

13) Find the distinct left cosets of H={0,4,8} in the group $(Z_{12}, +_{12})$.

14) Find the number of generators of a cyclic group of order 60.

15) Prove that every subgroup of an Abelian Group is Normal subgroup.

16) Define Normal Subgroup and give example.

17) Define Homomorphism of a Groups.

18) Define Kernal of Homomorphism of Groups.

19) State cayle's Theorem in Groups.

20) State lagrange's theorem in a finite Groups.

THREE Marks Questions

1) Evaluate $\iint_R xy(x + y)\, dxdy$ over the domain R between $y=x^2$ and y=x.

2) Prove that div(curl$\vec{F}$)=0.

3) Show that $(z_5, \oplus_5)$ is a group.

4) Evaluate $\int_c (3x-2y)dx + (y+2z)dy - x^2dz$, where c : x= t , y=$2t^2$, z=$3t^3$ and $0 \le t \le 1$.

5) Evaluate $\int_c (x+2y)dx + (4-2x)dy$, around the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$, in the counter clockwise.

6) Evaluate $\int_c \frac{yz}{x}dx + e^y dy + \sin z\, dx$, where c is the curve $x=t^3, y=t, z=t^2$, $2 \le t \le 3$.

7) Show that $\int_c y^2dx + 2xydy$ is independent of the path joining the points (0,1) and (1,3).

8) Evaluate$\int_0^1\int_1^{1-x} xy\, dydx$.

9) Find the directional derivative of φ =xy+yz+zx. In the direction of the vector 2i+j-k at the point (1,0,2). Also find maximum directional derivative.

10)Evaluate $\int_0^4\int_0^{\sqrt{4-x}} xy\, dydx$.

11) Evaluate $\int_0^1\int_0^{y^2} e^{\frac{x}{y}}\, dxdy$.

12) Evaluate $\int_0^1\int_0^1\int_0^1 (x+y-z)dxdydz$.

13) If $\vec{f} = x\hat{\imath} - y\hat{\jmath} + z\hat{k}$ then show that (a). div$\overrightarrow{f}$ =2 , (b) curl $\overrightarrow{f} = 0$

14) Prove that in a group G, o(a)=o(a^{-1}) $\forall a \epsilon G$.

15) If ‘n’ is any positive integer and ‘a’ is relatively prime to ‘n’, then $a^{\varphi(n)} \equiv 1(mod n)$

FIVE Marks Questions

<u>UNIT-1</u>

1) Evaluate $\iint_D (1+x^2+y^2)^{\frac{-1}{2}}$ dxdy, where D is the interior of lemniscate $r^2 = cos2\theta$ by changing into polar co-ordinates.

2) Evaluate $\int_0^1\int_{y^2}^{\sqrt{y}} \frac{y}{x}e^x\, dxdy$ by changing the order of integration.

3) Evaluate $\int_c (x^2+y^2)dx + x^3y\, dy$, where ‘c’ is the semicircle with centre at (0,4) and radius 2 units which lies in the first quadrant.

4) Evaluate $\int_0^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dydx$ by changing the order of integration.

5) Evaluate $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}}(x^2+y^2)$dydx by changing into polar co-ordinates.

6) Evaluate $\int_0^{a}\int_0^{\sqrt{a^2-x^2}}\int_0^{\sqrt{a^2-x^2-y^2}}\frac{dxdydz}{\sqrt{a^2-x^2-y^2-z^2}}$.

7) Evaluate $\int_0^{1}\int_0^{1-x}\int_0^{1-x-y}(xyz)dzdydx$.

8) Find the volume of the sphere $x^2+y^2+z^2=a^2$.

9) Evaluate $\iiint_R (x+y+z)$dxdydz. Where R is the region in the first octant bounded by the sphere $x^2+y^2+z^2=a^2$ by changing it to spherical polar co-ordinates.

10) Evaluate $\int_0^{\frac{\pi}{2}}\int_0^{a\sin\theta}\int_0^{\frac{a^2-r^2}{a}}(r)dzdrd\theta$.

UNIT-2

1) Find the equation of the tangent plane to the surface z=xy at (2,3,6). Also find the unit normal vector.
2) If $\vec{F}$ and $\vec{G}$ are any two vectors .Prove that $\nabla.(\vec{F}\times\vec{G})=\vec{G}\,.\,(\nabla\times\vec{F})-\vec{F}\,.\,(\nabla\times\vec{G})$
3) Find the scalar field φ such that $\nabla\varphi = y^2z^3\hat{\imath}+2xyz^3\hat{\jmath}+3xy^2z^2\hat{k}$ given that $\varphi(x,y,z)$=0 at the origin.
4) Find the directional derivative of $x^2y+z^2y-xz^3$ at (-1,2,1) also find maximum directional derivative .
5) Find the angle between the normal's to the surface $xy=z^2$ at the point (1,9,-3) and (-2,-2,2).
6) If $\vec{r}$ is the position of a point p (x,y,z) and $|\vec{r}\,|=r$, Show that $\nabla.(r^3\vec{r}\ )=6r^3$.
7) State and prove Green's theorem in the plane.
8) Using Green's theorem $\oint_c (3x^2-8y^2)dx+2y(2-3x)dy$ wher c is the boundary of the rectangular area enclosed by the lines $x=0, x=1, y=0\ and\ y=2$.
9) Using Gauss divergence theorem Evaluate $\iiint_v div(\vec{f})dv$ where $\vec{f}=2xy\hat{\imath}+yz^2\hat{\jmath}+xz\hat{k}$. And **s** is the total surface the rectangular parallelepiped bounded by $x=y=z=0\ \ and\ \ x=1, y=2, z=3$.
10) Evaluate by stoke's theorem $\oint_c yxdx+zxdy+xydz$, where c is the curve $x^2+y^2=1\,, z=y^2$.

UNIT-3

1) PT G={0,2,4,6,8} is an abelian group with respect to multiplicative modulo 10.
2) Prove that every subgroup of a cyclic group is cyclic.
3) Prove that a subset H of a group G is a subgroup of G if and only if $HH^{-1} = H$.
4) State and prove Lagrange's theorem on finite groups.
5) Prove that in a Group 'G' , ∀ a∈G , O(a)= O(a^{-1}).
6) Let G be a Group and a∈G . if O(a)= n and (m,n)=1 then O(a^m)= n.
7) Show that, if 'H' is a Subgroup of a group ' G' then H^{-1} = H. Is the converse true ?
8) Show that the Group G={1,-1,i-,i} is cyclic Group with respect to multiplication.
9) If **a** and ***x*** be any two elements of the group G then prove that $o(a) = o(xax^{-1})$.
10) If G be a cyclic Group of order **k** and **a** be a generator then Prove that if $a^m = a^n \ (m \neq n)$ then $m \equiv n(mod k)$ and conversely.

UNIT-4

1) Prove that a subgroup H of a group G is a normal subgroup of G iff gHg^{-1}=H $\forall g \in G$.
2) Prove that a subgroup H of a group G is a normal subgroup of G if fproduct of two right cosets of H in G is also a right coset of H in G
3) If H is a normal subgroup and K is normal subgroup G then Prove that $H \cap K$ is normal in H.
4) If G is a group and H is a subgroup of index 2 in G then Prove that H is normal subgroup of G.
5) If N is a normal subgroup of G and H is only subgroup of G, then Prove that NH is a subgroup of G.
6) If $f: G \to G'$ is homomorphism of groups with kernel K then Prove that K is normal subgroup of G.
7) State and Prove caylay Hamilton theorem.
8) If $f: G \to G'$ is homomorphism of groups with kernal K then PT f is one-one iff K={e}, where e belongs to G

9) If f be a homomorphism from group G to G' and G abelian then Show that G' is also abelian.

10) If $f: G \rightarrow G'$ is homomorphism of groups then Prove that

a) $f(e) = e'$ where $e \in G \; and \; e' \in G'$.

b) $f(a^{-1}) = [f(a)]^{-1} \; \forall \; a \in G$.

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