Introduction:- Game theory was developed by John von Newman. He worked on game theory right from 1928. But it gained prominence only after 1944 when he published (along with Morgenstern) the work Theory of games and economic be have our. This field study is fast developing and it is highly resourceful

Essential features of game theory:-

A competitive game has the following features

1) Finite number of competitions: - There are finite numbers of competitions called players.

The players used not be individuals, they can be groups corporations, political par ties institutions

2) Finite Number of action:- A list of finite number of possible courses of action is available to each player

3) Knowledge of Alternatives:- Each player has the knowledge of alternatives available to his opponent

4) Choice:- Each player makes a choice i.e. the games is played. The choices are assumed to be made simultaneously so that on player knows his opponents choice until he has decided his own course of action

5) Outcome or gain: - The play is associated with an outcome known as gain here the loss is considered negative gain.

6) Choice of opponent:- The possible gain or loss of each player depends upon not only the choice made by him but also the choice made by his opponent

Two persons zero-sum Games:- Two persons zero-sum game is situation which involve two persons or player and gains made by one person is equals to the less incurred by the other.

N= persons game:- A game involving ‘n’ persons is called a ‘n’ persons game. In this two persons game are most common when there are more players in a game are called ‘n’ persons game.

Pay offs:- Outcomes of a game due to adopting the different courses of actions by the competing players in the form of gains or losses for each of known as pay off.

Pay off Matrix: - In a game the strategy of a player is the predetermined rule by which he chooses his course of action while playing the game.

Types of strategy

There are two types of strategy

(1) **Pure strategy** (2) **Mixed strategy**

1) Pure strategy: - While playing a game pure strategy of a player is his predecision to adopt a specific course of action irrespective of the strategy of the opponent .

2) Mixed Strategy:- While playing a game mixed strategy of a player is his predecision to choose course of action according to certain preassined probabilities

Those if player A decides to adopt courses of action A_1 and A_2 with respective probabilities 0.4 and 0.6 it is mixed strategy.

THE MAXIMIN-MINIMAX PRINCIPLE:-

1) **Maximin criteria: -** The maximizing player lists his minimum gains from each strategy and select the strategy which gives the maximum out of these minimum gains.

2) **Minimax criteria:-** The minimizing player list his maximum loss from each strategy and select the strategy which gives him the minimum loss out of these maximum losses.

Types of problems:-

1) Games with pure strategy OR Two person zero sum game with saddle point:-

In case of pure strategy the game problems be solve a by using saddle point method or maximin & minimax rules

Saddle point:- The saddle point in a pay off mat x is one which is the smallest value in its row and the largest value in its column

The following steps are required of find out saddle point

1) Saddle the minimum value of each row & put circle ○ around it

2) Select the maximum value of each column and put square □ around it

3) The value with both circle & square $\boxed{\circ}$ is the saddle point & that is the value of the game

2) Games with mixed strategy without saddle point All game problems where saddle point does not exits are taken as mixed strategy problems where row minima is not equal to column maxima for the solution of game any of the following method

1) ODDS method (2×2 game without saddle point)

2) Dominance method

3) Sub games method-For ($m\times2$) or ($2\times n$) Matrices

4) Equal game method

5) Linear programming method-graphic solution & simplex method.

6) Itirative method

ODDS Method-For 2×2 Game

Use of odds method is possible only in case of game with 2×2 matrix , Here it should be ensured that sum of column odds and row odds is equal.

The ODDS method can be easily understand by the following table

$$
\begin{array}{cccccc}
 & & & Y & & \\
 & \text{Strategy} \rightarrow & y_1 & y_2 & & \text{ODDS} \\
 & \downarrow & & & & \\
X & x_1 & a_1 & a_2 & \rightarrow & (b_1-b_2) \\
 & x_2 & b_1 & b_2 & \rightarrow & (a_1-a_2) \\
 & & \downarrow & \downarrow & & \\
 & \text{ODDS} & (a_2-b_2) & (a_1-b_1) & &
\end{array}
$$

Note:- The above odds or differences are taken as positive (ignoring negative sign)

The value of game (v)= $\frac{a_1 (b_1-b_2)+b_1(a_1+a_2)}{(b_1-b_2) + (a_1-a_2)}$

Probabilities for $x_1 = \frac{b_1-b_2}{(b_1-b_2)+(a_1-a_2)}$ $x_2= \frac{(a_1-a_2)}{(b_1-b_2)+(a_1-a_2)}$

$Y_1= \frac{(a_2-b_2)}{(a_1-b_1)+ (a_2-b_2)}$ $y_2 = \frac{(a_1-b_1)}{(a_2-b_2)+(a_1-b_1)}$

Limitation of Game theory:-

1) **Infinite number strategy:-** In a game theory we assume that there is finite number of possible courses of action available to each player . But in practice a player may have infinite number of strategies.

2) **Knowledge about strategy**:- Game theory assume that each player has the knowledge of strategy available to his opponent. But sometimes knowledge about strategy about the opponent is not available to player. This leads to the wrong conclusion.

This leads to the wrong conclusion

3) **Zero out comes**: - We have assumed that gain of one person is the loss of another person But in practice gain of one person may not be equal to the loss of another person i. e opponent.

4**. Risk and Uncertainty**: - Game theory does not take into consideration the concept of probability. So game theory usually ignores the presence of risk and uncertainty.

5. Finite number of competitor:- there are finite number of competitors as has been assumed in the game theory but in real practice there can be more than the expected number of player.

6. Certainty of pay off:- Game theory assume that pay off is always known in advance. But sometimes it is impossible to know the pay off in advance. The decision situation in fact becomes multidimensional with large number of variables.

'Vogels method'

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Steps:-

1) For each row of the table identify the lowest & the next lowest cost cell. find their difference & place it to the right of that row in case two cells contains the same least cost then the difference shall be zero

2) Similarly find the difference of each column & place it below each column. these differences found in step-1 & 2 are also called penalties.

3) Looking up all the penalties identify highest of them & the row & the column relative to that penalty. allocate the maximum possible units to the least cost cell in the selected row or column.

4) Adjust the supply & Demand & cross the satisfied row or columns

5) Re-compute the column & row differences ignoring deleted row or columns & go to step no.3 repeat the procedure until all the column & row totals are satisfied

1) solve the following transportation problem by using vogels method.

	1	2	3	4	Supply
A	7	3	8	6	60
B	4	2	5	10	100
C	2	6	5	1	40
Demand	20	50	50	80	

Initial basic feasible solution by vogel's method.

	1	2	3	4	Supply	up1	up2	up3	up4
A	7	3 (20)	8	6 (40)	60	3	3	1 3	(4)
B	4 (20)	2 (30)	5 (50)	10	100	2	2	2	2
C	2	6	5	1 (40)	40	1	–	–	–
Demand	20	50	50	80	200/200				
up1	2	1	0	5↑					
up2	3	1	3	4↑					
up3	3	1	3	–					
up4	4	2	5↑	–					
up5	4↑	2	–	–					

Transportation cost = 3x20 + 6x40 + 4x20 + 2x30 + 5x50 + 1x40

= 60 + 240 + 80 + 60 + 250 + 40

= 730

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non-Degenerate Basic feasible solution = m+n-1

= 3+4-1

= 7-1

no of Allocation = 6

Rim requirement 6 = 6

(2) Solve the following transportation problem by using VAM method

		A	B	C	D	Availability
up5	Source I	21	16	25	13	11
(4)	II	17	18	14	23	13
	III	32	27	18	41	19
2	Requirement	6	10	12	15	43\43

	A	B	C	D	Availbality	up1	up2	up3	up4
I	21	16	25	13 (11)	11	3	–	–	–
II	17 (6)	18 (3)	14	23 (4)	13	3	3	3	4
III	32	27 (7)	18 (12)	41	19	9	9	9	9
Require	6	10	12	15	43\43				
up1	4	2	4	10↑					
up2	15	9	4	18↑					
up3	15↑	9	4	–					
up4	–	9↑	4	–					

NDBFS = m+n-1

= 3+4-1

= 7-1

= 6

Transportation cost = $13 \times 11 + 17 \times 6 + 18 \times 3 + 23 \times 4 + 27 \times 7 + 18 \times 12$
$= 143 + 102 + 54 + 92 + 189 + 216$
$= \underline{\underline{796}}$

③ Solve the following transportation problem by vogels method

	M_1	M_2	M_3	M_4	M_5	capacity
F1	4	2	3	2	6	8
F2	5	4	5	2	1	12
F3	6	5	4	7	3	14
Requirements	4	4	6	8	8	~~30~~ 34

	m_1	m_2	m_3	m_4	m_5	dummy	capa	up_1	up_2	up_3
F1	4	2 (4)	3	2 (4)	6	0	8	2	0	0
F2	5	4	5	2 (4)	1 (8)	0	12	1	1	2
F3	6 (4)	5	4 (6)	7	3	0 (4)	14	3 ←	1	1
Requi	4	4 ×	6	8	8 ×	4 ×	~~34~~ 34			
up_1	1	2	1	0	2	0				
up_2	1	2 ↑	1	0	2 ↑	–				
up_3	1 4	2 ↑	1	0	–	–				
up_4	1	–	1	(5) 0	–	–				
up_5	(6) 2	–	(4) 1	5 ↑	–	–				

$2 \times 4 = 8$

Transportation cost $= 2 \times 4 + 2 \times 4 + 1 \times 8 + 4 \times 6 + 6 \times 4 + 0 \times 4$
$8 + 8 + 8 + 24 + 24 + 0$
$= 80$

Transportation problem :-
unit-5

methods for initial basic feasible solution

Following are the important methods of deve-loping an initial feasible solution.

1) Northwest corner method (NWCM)
2) Lowest cost entry method (LCM)
3) Vogels method or vogels approximation method (VAM)
4) modi Method

I. North west corner method

① Solve the following by NWCM

	warehouse w_1	w_2	w_3	supply
plant P_1	7	6	9	20
P_2	5	7	3	28
P_3	4	5	8	17
Demand	21	25	19	65

Solⁿ:-

	warehouse w_1	w_2	w_3	supply
plant P_1	7 / 20	6	9	20
P_2	5 / 1	7 / 25	3 / 2	28
P_3	4	5	8 / 17	17
Demand	21	25	19	

Total transportation cost $= (20\times7)+(5\times1)+(7\times25)+(3\times2)$
$+ (8\times17)$
$= 140+5+175+6+136$
$= \underline{462}$

② Solve the following transportation problem of NWCM. 2016

From \ To	w_1	w_2	w_3	w_4	supply
F1	30	25	40	20	100
F2	29	26	35	40	250
F3	31	33	37	30	150
Demand	90	160	200	50	500

Solⁿ

	w_1	w_2	w_3	w_4	supply
F1	30 / 90	25 / 10	40	20	100
F2	29	26 / 150	35 / 100	40	250
F3	31	33	37 / 100	30 / 50	150
Demand	90	160	200	50	

Total transportation cost =
$(30\times90)+(25\times10)+(26\times150)+(35\times100)+$
$(37\times100)+(30\times50) =$
$\underline{15550}$

(3) Solve the following problem by NWCM

~~W1 W2 W3 W4 supply~~

mkt / plant	A	B	C	plant supply
X	11	21	16	14
Y	7	17	13	26
Z	11	23	21	36
mkt Requirement	18	28	25	~~71~~ 76

Soln

market / plant	A	B	C	Dummy	supply
X	11 (14)	21	16	0	14
Y	7 (4)	17 (22)	13	0	26
Z	11	23 (6)	21 (25)	0 (5)	36
Requirement	18	28	25	05	~~71~~ 76

Total transportation cost

$= 11\times14 + 7\times4 + 17\times22 + 23\times6 + 21\times25 + 0\times5$

$= 154 + 28 + 374 + 138 + 525 + 0$

$= \underline{\underline{1219}}$

(4) Solve the following transportation problem by NWCM

Warehouse / Factory	W_1	W_2	W_3	W_4	capacity
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
Requirement	5	8	7	14	34

calculation of transportation cost by NWCM

	W_1	W_2	W_3	W_4	capacity
F_1	$\boxed{19}$ 5	$\boxed{30}$ 2	$\boxed{50}$	$\boxed{10}$	7
F_2	$\boxed{70}$	$\boxed{30}$ 6	$\boxed{40}$ 3	$\boxed{60}$	9
F_3	$\boxed{40}$	$\boxed{8}$	$\boxed{70}$ 4	$\boxed{20}$ 14	18
Requirement	5	8	7	14	34

Total transportation cost

$= 19 \times 5 + 30 \times 2 + 30 \times 6 + 40 \times 3 + 70 \times 4 + 20 \times 14$

$= 95 + 60 + 180 + 120 + 280 + 280$

$= \underline{1015}$

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⑤ Solve the following transportation problem by NWCM.

D \ W	D_1	D_2	D_3	D_4	D_5	Availability
W_1	3	4	6	8	8	20
W_2	2	10	0	5	8	30
W_3	7	11	20	40	3	15
W_4	1	0	9	14	16	13
Requirement	40	6	8	18	6	78

calculation of transportation cost by NWCM

	D_1	D_2	D_3	D_4	D_5	Availabo
W_1	3 (20)	4	6	8	8	20
W_2	2 (20)	10 (6)	0 (4)	5	8	30
W_3	7	11	20 (4)	40 (11)	3	15
W_4	1	0	9	14 (7)	16 (6)	13
Requirement	40	6	8	18	6	78

Total transportation cost

$= 3\times20 + 2\times20 + 10\times6 + 0\times4 + 20\times4 + 40\times11 + 14\times7 + 16\times6$

$= 60 + 40 + 60 + 0 + 80 +$

874//

⑥ Find the feasible solution of the following transportation problem by NWCM

F\W	w_1	w_2	w_3	w_4	supply
F1	14	25	45	5	6
F2	65	25	35	55	8
F3	35	3	65	15	16
Requirement	4	7	6	13	30 \ 30

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II Least cost entry method :-

(or)

matrix minimum method

(1) Solve the following transportation problem by matrix minimum method.

warehouse

		w_1	w_2	w_3	supply
plant	p_1	7	6	9	20
	p_2	5	7	3	28
	p_3	4	5	8	17
Demand		21	25	19	65

Sol^n

warehouse

		w_1	w_2	w_3	supply
plant	p_1	7	6 (20)	9	20
	p_2	5 (4)	7 (5)	3 (19)	28
	p_3	4 (17)	5	8	17
Demand		21	25	19	65

Total transportation cost

$= 6\times20 + 5\times4 + 7\times5 + 3\times19 + 4\times17$

$= 300$

(2) Solve the following transportation problem by LCEM.

warehouse

		A	B	C	supply
plant	X	50	30	220	41
	Y	90	45	170	13
	Z	250	200	50	4
Demand		13	23	22	58

Soln:-

warehouse

	A	B	C	supply
plant X	50 (13)	30 (23)	220 (5)	41
Y	90	45	170 (13)	13
Z	250	200	50 (4)	4
Demand	13	23	22	58

Total transportation cost $= 50\times13 + 30\times23 + 220\times5 + 170\times13 + 50\times4$

$= 650 + 690 + 1100 + 2210 + 200$

$= \underline{\underline{4850}}$

(3) solve the following problem with least cost entry method

warehouse

plant supply	A	B	C	supply
X	11	21	16	14
Y	7	17	13	26
Z	11	23	21	36
Requirement	18	28	25	~~71~~ 76

Note:- given problem is unbalanced transportation problem it should be converted balanced transportation problem by creating dummy problem.

	A	B	C	D (dummy)	supply
plant X	11	21	16 (14)	0	14
Y	7 (18)	17	13 (8)	0	26
Z	11	23 (28)	21 (3)	0 (5)	36
Requirement	18	28	25	5	76

$= 16 \times 14 + 7 \times 18 + 13 \times 8 + 23 \times 28 + 21 \times 3 + 0 \times 5$

$= \underline{\underline{1161}}$

(4) Q. Solve the following transportation problem by LCEM.

Factory	D_1	D_2	D_3	D_4	Available
F_1	1	2	4	4	8
F_2	4	3	2	0	6
F_3	0	2	2	1	10
Required	4	6	8	6	24

Solⁿ

	D_1	D_2	D_3	D_4	Available
F1	1	2 / 6	4 / 2	4	8
F2	4	3	2	0 / 6	6
F3	0 / 4	2	2 / 6	1	10
Requirement	4	6	8	6	24

$2 \times 6 + 4 \times 2 + 0 \times 6 + 2 \times 6 + 0 \times 4$

$12 + 8 + 0 + 12 + 0$

$\underline{\underline{32}}$

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